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Abstract

This document contains the modeling specifications for the EURACE Goods, Labour and Credit Markets.

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1 Introduction

This document contains the modeling specifications for the goods, labour and credit markets for the EURACE project, which aims at developing an agent-based platform for European economic policy design, applying a bottom-up approach in order to build a micro-founded model of the European economy. This model will eventually allow to test the effect of macroeconomic policy in an economic system composed by a large number of interacting heterogenous agents.

In this paper we will describe in detail the model for the credit market and the entry & exit process of firms. Consumption goods and labor markets are treated in detail in Bielefeld Unit deliverable D7.1: therefore, we will only offer a quick overview of how they work, and on the main modelling choices that have been made.

We proceed in the following way. In the next two sections we give a theoretical motivation for the use of agent-based modeling and an overview of the general features of the EURACE model. In section 4 we introduce an empirical analysis of bank-firm networks, based on a sample of italian banks, in order to get useful insights on the firm-bank relationship. On the basis of the previous empirical results, a theoretical framework for the banking sector is then modelled in section 5. In section 6 a simple mechanism for the entry & exit process is proposed. Finally, in section 7 we sketch a simple version of EURACE model, whose simulation results show an high degree of statistical precision compared to real data. In appendix, we report some stylized facts not related to credit markets.

2 Why agent-based models

2.1 Introduction

The methodological basis which EURACE model is grounded on, originate from the need to overcome the theoretical and empirical weaknesses of neoclassical approach to macroeconomics.

The battleground we choose for our study is the research program launched some forty years ago by the neoclassical school, according to which macroeconomics should be explicitly grounded on microfoundations. Briefly, economic phenomena at a macroscopic level should be explained as the consequences of the activities undertaken by individual decision makers. The methodological strategy that has so far gained supremacy is one based on:

- (i) the precepts of the modern (i.e., rational choice-theoretic) tradition;
- (ii) a solution (or *closing*) concept borrowed from Walrasian competitive general equilibrium analysis.

We endorse the accusation repeatedly brought against modern (*neoclassical*) economic theory (known also as *efficient resource allocation theory*) to be empirically and logically flawed¹. Some of the key flaws underlying item (i) has been presented in Keen (2003). In turn, critical words against item (ii) has been loudly pronounced, among the others, by eminent scholars who spent most of their academic life in the neoclassical camp (Hahn and Solow, 1995). Admittedly, their criticism was at one time well centered and largely neglected, probably because the alternative methodology they suggested was so close to the mainstream that the disturbing theoretical results they presented were seen as a nuisance, which could be easily addressed by painless manipulations of the standard model.

The alternative proposal we shall confront with the Walrasian paradigm in what follows is definitely more radical. As a preliminary step, however, it seems worthwhile to review why Walrasian microfoundations should be considered as the wrong answer to what is probably the most stimulating research question ever raised in economics, that is to explain how a completely decentralized economy composed of millions of (mainly) self-interested people coordinate their actions.

2.2 From Smith to neoclassical reductionism

Contrary to what is usually thought, the very idea that the economy is a (complex) self-organizing system is not a new entry in the toolbox of economists got mixed up in complexity, but it is the key message conveyed in 1776 by the founding father of the discipline, the Scottish moral philosopher Adam Smith, according to whom:

“He [man] generally, indeed, neither intends to promote the public interest, nor knows how much he is promoting it. By preferring the support of domestic to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention.

¹Neoclassical economics is axiomatic. While it requires internal coherence, so that theorems can be logically deduced from a set of assumptions, it abstracts from external coherence between theoretical statements and empirical evidence. Of course, this implies an important epistemological detachment from falsifiable sciences like physics.

In civilized society he [man] stands at all times in need of the cooperation and assistance of great multitudes, while his whole life is scarce sufficient to gain the friendship of a few persons. In almost every other race of animals each individual, when it is grown up to maturity, is entirely independent, and in its natural state has occasion for the assistance of no other living creature. But man has almost constant occasion for the help of his brethren, and it is in vain for him to expect it from their benevolence only. He will be more likely to prevail if he can interest their self-love in his favour, and show them that it is for their own advantage to do for him what he requires of them. Whoever offers to another a bargain of any kind, proposes to do this. Give me that which I want, and you shall have this which you want, is the meaning of every offer; and it is in this manner that we obtain from one another the far greater part of those good offices which we stand in need of. It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their self-love, and never talk to them of our own necessities but of their advantages” (Smith, 1776, p. 477 of the 1976 University of Chicago edition).

It is clear that Smith’s vision of real economies is close to that of a complex system, produced by the inter-actions of a multitude of heterogeneous agents. Five points deserve to be emphasized. First, the notion of an *invisible hand* guiding coordination towards aggregate order in a fully decentralized economy represents an *explanandum* (i.e., a research question) of theoretical and empirical importance, both for positive and normative reasons. Second, the explanation of economic phenomena is run in terms of the actions and reactions of autonomous individuals and not of social categories. This requires a conceptualization of decision-makers in terms of *economic agents*, rather than social classes or races. Third, the economic agent (Smith’s *man*) envisaged in the quotation above responds to incentives to improve utility, but he is never requested to maximize it as he should do if endowed with substantive rationality. Fourth, Smith recognizes self-interest as being the main force driving decentralized actors towards a coordinated aggregate position, but he never concedes that greed is the only relevant human trait. Fifth, the work of the invisible hand yields a social order, but neither is such an order a rest (i.e., an equilibrium) nor is the best possible solution (i.e., an optimum).

As regards the first two points –the key challenge of explaining how the invisible hand works and the approach of methodological individualism², respectively– the profession has reached a consensus to which we convincingly adhere. Moving to the last three points, however, we subscribe to the far less popular claiming that the theoretical approach developed by the mainstream in dealing with them has proven to be inadequate. Instead of the idea of a self-regulating order emerging from the interactions of many simpler components without this *being part of their intentions*, the Holy Trinity of (marginalist) neoclassical economics (Colander, 2005) –(substantive) rationality, greed and equilibrium– has soon become the elected guiding principles of human behavior. In

²Actually, the path followed by methodological individualism has been rather bumpy, and what is nowadays commonly accepted by its very notion is somewhat different from the origins. After having brilliantly repelled the attack of Marxians, the currently established version of methodological individualism is illustrated by Arrow (1994), who acknowledges that individual behavior is always mediated by social relations, but also that social relations are the outcome of the actions of individual agents.

setting the methodological stage for the revolution dubbed Dynamic Stochastic General Equilibrium (DSGE) macroeconomic theory, Robert Lucas and Thomas Sargent bluntly declared: “An economy following a multivariate stochastic process is now routinely described as being in equilibrium, by which is meant nothing more than that at each point in time (a) markets clear and (b) agents act in their own self-interest. This development, which stemmed mainly from the work of K. J. Arrow [...] and G. Debreu [...], implies that simply to look at any economic time series and conclude that it is a disequilibrium phenomenon is a meaningless observation. [...] The key elements of these models are that agents are rational, reacting to policy changes in a way which is in their best interests privately, and that the impulses which trigger business fluctuations are mainly unanticipated shocks. (Lucas and Sargent, 1979, p. 7.)

The self-regulating order of Smith was therefore transformed into a competitive General Equilibrium (GE) in the form elaborated in the 1870s by Leon Walras (1874), that is a configuration of (fully flexible) prices and plans of action such that, at those prices, all agents can carry out their chosen plans and, consequently, markets clear. In a continuous effort of generalization and analytical sophistication, modern (neoclassical) economists interested in building microfoundations for macroeconomics soon recurred to the refinement proposed in the 1950s by Arrow and Debreu (1954), who showed that also individual *intertemporal* (on an infinite horizon) optimization yields a GE, as soon as the economy is equipped with perfect price foresights for each future state of nature and a complete set of Arrow-securities markets (1964), all open at time zero and closed simultaneously. Whenever these conditions hold true, the GE is an allocation that maximizes a properly defined social welfare function or, in other terms, the equilibrium is Pareto-efficient (First Welfare Theorem)³. As already anticipated, the weaknesses of the epistemological status of the GE model are so deep to have been fully recognized at various stages by its very proponents. Since this awareness still encounters huge difficulties in being spread among the profession, however, it seems worthwhile to provide a concise exposition of the main issues at hand.

1. The GE is neither unique nor locally stable under general conditions. This negative result, which refers to the work of Sonnenschein (1972), Debreu (1974) and Mantel (1974), can be summarized along the following lines. Let the aggregate excess demand function $F(p)$ –obtained from aggregating among individual excess demands $f(p)$ – be a mapping from the price simplex Π to the commodity space P^N . A GE is defined as a price vector p^* such that $F(p^*) = 0$. It turns out that the only conditions that $F(\cdot)$ inherits from $f(\cdot)$ are continuity, homogeneity of degree zero and the Walras law (i.e., the total value of excess demand is zero). These assure the existence, but neither the uniqueness nor the local stability of p^* , unless preferences generating individual demand functions are restricted to very special cases.
2. The existence of a GE is proved *via* the Brouwer fix point theorem, i.e. by finding a continuous function $g(\cdot) : \Pi \rightarrow \Pi$ such that any fix point for $g(\cdot)$ is also an equilibrium price vector $F(p^*) = 0$. Suppose that we are interested in finding an

³See Ingrao and Israel (1990) for a remarkable account of the origin and evolution of the GE concept.

algorithm, which, starting from an arbitrary price vector p , chooses price sequences to check for p^* and halt when it finds it. In other terms, to find the GE price vector $F(p^*) = 0$ means that halting configurations are decidable. As this violates the undecidability of the halting problem for Turing Machines, from a recursion theoretic viewpoint the GE solution is uncomputable (Richter and Wong, 1999; Velupillai, 2000). Notice that the same problem applies, in spite of its name, to the class of *Computable* GE models (Velupillai, 2005).

3. By construction, in a GE all transactions are undertaken at the same equilibrium price vector. Economic theory has worked out two mechanisms capable to reach this outcome. First, one can assume that buyers and sellers adjust costlessly their optimal supplies and demands to prices called out by a (explicit or implicit) fictitious auctioneer, who continues to do his job until he finds a price vector which clears all markets. Only then transactions take place (Walras assumption). Alternatively, buyers and sellers sign provisional contracts and are allowed to freely (i.e., without any cost) re- contract until a price vector is found which makes individual plans fully compatible. Once again, transactions occur only after the equilibrium price vector has been established (Edgeworths assumption). Regardless of the mechanism one adopts, the GE model is one in which the formation of prices precedes the process of exchange, instead of being the result of it, through a *tatonnement* process occurring in a meta-time. Real markets work the other way round and operates in real time, so that the GE model cannot be considered a scientific explanation of real economic phenomena (Arrow, 1959).
4. It has been widely recognized since Debreu (1959) that integrating money in the theory of value represented by the GE model is at best problematic. No economic agent can individually decide to monetize alone; monetary trade should be the equilibrium outcome of market interactions among optimizing agents. The use of money –that is, a common medium of exchange and a store of value– implies that one party to a transaction gives up something valuable (for instance, his endowment or production) for something inherently useless (a fiduciary token for which he has no immediate use) in the hope of advantageously re-trading it in the future. Given that in a GE model actual transactions take place only after a price vector coordinating all trading plans has been freely found, money can be consistently introduced into the picture only if the logical keystone of the absence of transaction costs is abandoned. By the same token, since credit makes sense only if agents can sign contracts in which one side promises future delivery of goods or services to the other one, in equilibrium markets for debt are meaningless, both information conditions and information processing requirements are not properly defined, and bankruptcy can be safely ignored. Finally, as the very notion of a GE implies that all transactions occur only when individual plans are mutually compatible, and this has to be true also in the labor market, the empirically observed phenomenon of involuntary unemployment and the microfoundation program put forth by Lucas and Sargent are logically inconsistent.
5. The only role assigned to time in a GE model is that of dating commodities.

Products, technologies and preferences are exogenously given and fixed from the outset. The convenient implication of banning out-of-equilibrium transactions is simply that of getting rid of any disturbing influence of intermediary modifications of endowments –and therefore of individual excess demands– on the final equilibrium outcome.

The introduction of non-Walrasian elements into the GE microfoundations program –such as fixed or sticky prices, imperfect competition and incomplete markets leading to temporary equilibrium models– yields interesting Keynesian features such as the breaking of the Says law and scope for a monetary theory of production, a rationale for financial institutions and a more persuasive treatment of informational frictions. As argued in Vriend (1994), however, all these approaches preserve a Walrasian perspective in that models are invariably closed by a GE solution concept which, implicitly or (more often) not, implies the existence of a fictitious auctioneer who processes information, calculates equilibrium prices and quantities, and regulates transactions. As a result, if the Walrasian Auctioneer (WA) is removed the decentralized economy becomes dynamically incomplete, as we are not left with any mechanism determining how quantities and prices are set and how exchanges occur.

The solution adopted by mainstream macroeconomists to overcome the problems of uniqueness and stability of equilibrium on the one hand, and of analytically tractability on the other one is the usage of a *Representative Agent* (RA), whose choices summarize those of the whole population of agents. The flaws of this solution are so pervasive and well known that it seems worthless to discuss them here (see for example Kirman (1992), Hartley (1997) and Gallegati *et al.* (2006)), and we proceed to present an alternative solution to the microfoundation issue.

2.3 A constructive approach to macroeconomics

The research methodology we endorse in trying to explain successes and failures of the invisible hand, and which will be at the *core* of EURACE model, consists in discarding the Walrasian GE approach to the microfoundation program, as well as its RA shorthand version. Instead of asking to deductively *prove* the existence of an equilibrium price vector p^* such that $F(p^*) = 0$, we aimed at explicitly constructing it by means of an algorithm or rule. From an epistemological perspective, this implies a shift from the realm of classical to that of constructive theorizing [28]⁴. Clearly, the act of computationally constructing a (fully or not) coordinated state –instead of imposing it *via* the WA– for a decentralized economic system requires a complete description of goal-directed economic agents and their interaction structure.

Agent-based computational economics (ACE) –that is the use of computer simulations to study evolving complex systems composed of many autonomous interacting agents– represents an effective implementation of such a research agenda (Judd and Tesfatsion, 2006). ACE allows an explicit modeling of identifiable, goal-directed, adapting agents, situated in an explicit space and interacting locally in it. In complex adaptive systems

⁴Epstein (in [29]) prefers to talk of a generative approach to scientific explanation, but the meaning is basically the same.

local interactions involves the spontaneous formation of macroscopic structures which can not be directly deduced by looking at individual behaviors. The equilibrium of a system does not require any more that every single element is in equilibrium by itself, but rather that the statistical distributions describing aggregate phenomena are stable, i.e. in [...] *a state of macroscopic equilibrium maintained by a large number of transitions in opposite directions* (Feller, 1957, p. 356). A consequence of the idea that macroscopic phenomena can emerge is that the strong reductionist vision retained by neoclassicals is basically wrong. Once again, these concepts should be familiar to economists, at least to those who pay attention to the history of economic thought, since they mirror the notion of *spontaneous market order* or *catallaxy* put forth by the Nobel Prize in economics Friedrich von Hayek. According to him, a clear definition of the laws of property, tort and contract is enough to regulate a set of trial and error exchange relationships, which succeeds in coordinating the plans of an interdependent network of individuals endowed with a multiplicity of competing ends. In contrast, Hayek argues that the notion of competitive GE is *unfortunate, since it presupposes that the facts have already all been discovered and competition, therefore, has ceased* (Hayek, 1978, p. 184).⁵

3 General features of EURACE model

The EURACE model is a closed model composed by a consumption good market, a capital goods market, a labour market, a credit and a financial market.

From a general point of view, in order to build an agent-based model four main points needs to be addressed:

- the list of the agents that populate the model. Being the credit market only a part of the entire economic system, most of the agents will be the same that populate the other markets: the firms, both of the capital and of the consumption good market and the consumers. In this model we introduce a new kind of agent: the banks, that can be considered as a third kind of firm.
- the structure of the agents. It is composed by:
 1. a list of the variables that describe each single agent in every step of the simulation. While the internal state of the capital and consumption good producing firms and of the consumers are described in other papers, here we are mainly concerned with the variables that describe the internal state of the banks.

⁵It must be emphasized, however, that the abandonment of the GE solution concept does not come at no cost. The optimality of a competitive GE associated with the First Welfare Theorem implies that welfare comparisons between alternative macroeconomic states can be meaningfully carried out. An operational method for performing aggregate welfare comparisons also in macro ACE models has been advanced by Tesfatsion (2006). The idea consists in defining an ideal benchmark (for instance, the GDP in correspondence of continual market clearing) and comparing it to the simulation outcomes. We can say the system has reached a catallaxy if the distance between the simulated time series for GDP and the ideal reference path differs by less than a tolerance level τ .

2. a list of the possible actions the agents can perform and that have an effect on their internal state and in those of other agents.
- the interaction network that links the various kind of agents. Of course in a model of the economic system such as this, we will have many different and overlying networks: the interaction networks that links producers to consumers, the interaction network that links the firms to the workers, and so on. In this paper we focus on two interaction networks that links the banks with the economic system, that is, the network that links the banks to the agents that need external financial resources and the network that link the banks to those agents who save part of their income, and who decide to deposit these savings in the banks. Strictly linked to the definition of the structure of the interaction networks is the definition of the nature of the interaction itself. We can have two kind of interaction:
 1. direct interactions: are those interactions where the action of an agent directly affects the internal state of the agents with whom he is linked (for example, the firm that fires a worker). In particular, in the credit market we have direct interaction when the agents decide to deposit financial resources in the banks and when the banks decide to accept the agents' requests for credit.
 2. indirect interactions: are those interactions that give place to an information exchange among the agents (for example, in the consumer good market, the information exchange about the price of a product between the firm and the consumer). Of course many interactions produce both a direct effect and an information exchange between the involved agents. In the credit market the information exchange is even more important than in the other markets, as the literature on asymmetrical information shows. From this point of view, it is important here to define the flux of information that takes place between the banks and the agents that apply for a loan.
 - the cognitive and behavioural model of the agents. It is the model of the decision process that take place inside the agents and that links their internal states and the information they can access at their decision about the action to undertake. Agent-based models are based on the bounded rationality paradigm according to which "models of human judgement and decision-making should be built on what we actually know about mind's capacities. Because of the mind's limitations humans must use approximate methods to handle most tasks" (Simon, 1990). So, we depart from the strong assumptions on the rationality of individuals that underlie much of the mainstream economics, trying to adopt more realistic models of the way economic agents take decisions. Deviation from the maximization paradigm opens many degrees of freedom with respect to the type of behavioural rules adopted by the agents and the process through which they are adapted over time. In developing the model of the credit market our strategy, coherently with the approach adopted for the other markets, has been to implement relatively simple decision rules that match standard procedures of the real world agents

involved in credit transactions. Of course, the cognitive and behavioural aspects are strictly connected to other aspects of the model. In particular, the agent's decision process is dependent on inputs coming from the interaction networks that link the agent to his environment, while the outcome is a decision on which action to undertake, based on the expected effects it will have on the internal variables of the agent itself. So, it is evident how crucial is it to maintain a logical and coherent philosophy throughout the development of all the various aspects of the model of the agents.

Of course, once the model has been designed and implemented, an important part of the development process is the validation of the model itself. In our view, the validation should not be viewed as the final stage according to which accept or discard the model, but a stage of a feed-back process that allows to calibrate the model. We have to note that the aim of an agent-based model is not, and could not be, the forecast of the dynamics of economic macro-variables, like those produced by econometric models, but the reproduction of their statistical properties. So, in the validation stage, the qualitative patterns of simulation result will have to be confronted with the stylized facts that have been discovered using real world data.

The EURACE model consists of a capital good, a consumption good, a labour, a credit and a financial market. We consider the energy market as exogenous and, accordingly, we fix exogenously the energy prices that affects the production costs of the capital goods producing firms. The whole economy consists of R regions and each agent is located in one of these regions, which are organized in a rectangular grid, determining the neighbourhood structure of the economy. In the model there are both the interactions which will take place only locally between the agents located inside same region (as for the labour market), and the interactions which take place globally, as in the investment good market. The minimal time unit in the EURACE model is the day, but some actions will occur only on weekly or monthly basis. In particular, the decision that will determine the supply and demand of financial resources, such as the decisions regarding the amount of income to save and those regarding the investments, are taken once a month.

In what follows, we will focus on the credit market. Nevertheless, it is useful to briefly summarize the whole model in order to give a comprehensive view of the interconnections between the different markets. In the full EURACE model, these markets are present as agent-based models and interact with each other through interfaces. The labour market is populated with workers which have a finite number of general skill levels and acquire specific skills on-the-job. They are used both in the good markets as a fundamental production factor. In the capital good market we have a small number of investment good producers that have a constant coefficient, constant returns to scale production function, which depends solely on labour and energy. Each of them produces an investment good that is based on a particular technology that can be improved by the means of process innovations that are the result of investments in R&D. While for the production these firms use workers that have low general skills, in the R&D department they employ workers with the highest skill level.

Downstream we have many firms producing vertically differentiated consumption goods, whose quality can be increased over time by means of investments in product

innovation. Consumption goods are produced with specifically skilled labour and with investments goods that can be bought in the capital goods market, from the investment good producers. Like the investment good producers, the consumption goods producing firms employ in the R&D division only the workers with the highest skill level. The consumption goods are sold in local markets.

Firms producing consumer goods may finance their expenditures for labour and capital investments with own revenues, savings, bank credits or in the financial market. If the debt to revenue ratio of a firm exceeds a certain threshold the firm goes bankrupt and exits the market. Consequently, banks will incur in losses.

Once a month, when they receive their income, consumers allocate their cash at hand to savings and consumption. It is assumed that every consumer visits once a week one local market to purchase consumer goods. Consumers first decide which local market to visit, then they sample qualities, prices and available stocks of the offered goods and make their purchasing decisions based on this information and the budget they have allocated for consumption.

3.1 The Consumption Goods Market

Consumption goods producers (GCP) need physical capital and labor to start production. Each CGP uses a capital good of a particular technology J_{it} , which is the cumulation of past investments and depreciates at rate δ . The average productivity of individual capital stock is given by the average quality of the stock of investment goods. Together with capital, GCP uses labor, whose productivity is given by the average skills of firm's workers. Workers have two kinds of skills: general and specific, the latter acquired with job experience. Hence, total production will depend both on average productivity of labor and capital, and on their total stocks.

CGP are also buyers on capital goods market, and their actions are affected by three aspects: (*i*) they need to decide which particular technology to adopt; (*ii*) they need to determine the variation (investment) of labor and capital stock; (*iii*) they have to determine the actual production quantity of the consumption good on the basis of the adjustment of the stocks (point *ii*).

The technology choice is taken on cost saving basis, whereas production level choice will depend on profits a firm expects to gain in a particular mall-region.

3.2 The Labour Market

Labor market is based on an individual random searching process between workers and firms. Labor demand is determined by capital and consumption good firms according to their production levels.

Both types of producers hire only high generally skilled workers in order to carry out R&D activity. As far as production is concerned, capital producers hire only low generally skilled workers, whereas consumption goods firms look also for specific skilled workers.

Labor supply consists of unemployed people plus a fraction of employed workers willing to change job. Job-seekers do not distinguish between the kinds of firms, but look at the

wage offer. Only if offered wage is higher than worker's reservation wage, then the worker applies to the firm. Firms post vacancies that stay valid for one day only, and rank in a list applying workers on the basis of skill levels. Best workers are then contacted by each firm; finally, workers accept their best job offer. Firms left with some vacancy open will raise their wage bid, while unemployed workers lower their reservation wage.

3.3 The Credit Market

In order to develop a credit market, some requirements to guide the model specification must be laid down. This is done on the basis of an empirical analysis that is described in the next section. The description of the credit market in the *EURACE* model is thus postponed to section 5.

4 Empirical analysis of bank-firm networks

A crucial point in the study of agent-based models in Economics is to identify the main economic mechanisms that reproduces the empirical observations.

In particular, the analysis of the empirical emerging architecture of relationships is of fundamental importance in order to understand which are the most important economic relationships among all the possible interactions. The empirical analysis of the observed credit relationships is a starting point for the theoretical models but also their final benchmark: a model that reproduces the empirical architecture is highly probably a model that captures the fundamental ingredients to represent the economic system. Part of the results will be directly implemented in the *EURACE* credit market scheme in order to reduce the number of freedom degrees, while a part is supposed to emerge spontaneously from the local interactions between banks and firms.

In the next subsection we sketch theoretical motivations for the following empirical research.

4.1 Theoretical motivations

Asymmetric information theory deeply affected economic discipline. The basic mainstream paradigm in economic theory - Arrow-Debreu version of the general equilibrium model- is one that imperfect information directly challenges: the individuals take decisions in isolation, using only information received through some general market signals such as prices.

Macroeconomists tried to avoid the problem of uniqueness and stability of equilibrium of the general equilibrium model by resorting to what has become the standard paradigm in modern macroeconomics, that is the *representative agent* framework, where the aggregate is supposed to match with individual behavior. However, under general conditions, this conjecture has been proved to be fallacious (Kirman, 1992): in fact, it means that one has to ignore communication and direct interaction among agents, which, ultimately, defines away the problem of coordination (Leijonhufvud, 1992; Hahn

and Solow, 1995): once again, in the general equilibrium model interaction and coordination occur only through prices.

The role of prices is undoubtedly important, but the price mechanism alone can work only if information is complete; in such a case, one can ignore the influence of other coordination and interaction mechanisms. In the literature on imperfect information two aspects are shown to be fundamental: (i) agents are heterogeneous and, (ii), markets are incomplete, i.e. the future is uncertain. If (i) and (ii) hold true, *interaction among agents* and *credit among agents and periods of time* become constitutive elements of the picture. Central to this query is information: about the economic status of each party active in the transaction, but also about the incentive structures that the party faces. Every time, the party providing the credit has to ascertain both the risks involved and their ability and willingness to bear the risk. In effect, all of the questions crucial to modeling banks' supply of credit are relevant in determining the supply of credit by firms, households, or non-bank financial institutions. All in all, direct, non-price interaction happens through the credit channels.

In the model these hypotheses will be taken seriously by considering the possibility of bankruptcies. In fact, in a financially imperfect world where informative imperfection is pervasive, unexpected events on future revenues may affect bank-firm relation as well as inter-firms credit link. In other words, we will model (possibly) an *inside credit* (*commercial credit*) between firms of different productive sectors, and an *outside credit* (*bank credit*), which is provided by banks, lending credit to the industrial sector (Stiglitz and Greenwald, 2003).

A central determinant of the level of economic activity is the supply of credit, and the most important institution in determining it is the banking system. Whenever an economic exchange without an immediate flow of money occurs, a credit-debt relationship emerges. Failure of fulfilling debt commitments could lead to *bankruptcy chains*. If debt commitments are not fulfilled, *bad debt* increases with the likely consequence of an increase in the interest rate. This latter leads to more bankruptcies: "*the high rate of bankruptcy is a consequence of the high interest rate as much as a cause of it*" (Stiglitz and Greenwald, 2003: 145). Thus, domino effects can arise through the credit channel, that is, an avalanche of bankruptcies may be caused by the diffusion of negative externalities in the network of bank-firm relationships.

At this point, the role of interconnections between banks and firms becomes clear. In what follows we present a set of empirical results obtained by means of network analysis.

4.2 Network theory

The network theory is the most natural mathematical environment to investigate the architecture of relationships in an economic context because it allows to consider explicitly the interactions among economic agents.

A network is defined as a set of nodes and links: it is mathematically represented as a graph. In the recent years there was a large development of the complex networks theory (Dorogovtsev et al., 2003; Caldarelli, 2006); in fact, many real systems can, and actually have been, represented as networks. This approach allows to get insights into the architecture of complex systems composed by many interacting objects/agents,

linked to each other in a non trivial way. The individual objects are represented by nodes and the interactions by links. The number of agents each agent is linked to (i.e. the number of links) is called the degree.

Most of the real networks are shown to be different from random graph⁶ and show peculiar scaling properties. In particular many networks are scale-free, which means that the degree distribution is power-law. This kind of network has been extensively studied since the seminal work of Barabasi in 1999 (Barabási et al., 1999): its topology is proven to be more resilient to random failures (Albert et al., 2000) maintaining a strong connectedness but more exposed to virus contagion (Pastor-Satorras et al., 2001). In the recent years this kind of approach is increasingly often applied to economic problems. Many empirical analysis of economic systems have be done from the world trade web (Garlaschelli et al., 2004; Serrano et al., 2003), to the interbank market (Iori et al., 2007), from the stock market (Garlaschelli et al., 2004b), to the e-commerce (Reichardt et al., 2005). In this context we use the network theory to better investigate the architecture of debt/credit relationships in the credit market. In particular we focus on the Italian credit market.

Among European countries, Italy has the biggest average number of bank relationships per firm (Ongena et al., 2000a; Ongena et al., 2000b). The multiple bank relationships became more relevant in Italy in the 50s. In fact after the Second World War Italian firms were able to finance themselves: therefore the interaction among banks and firms was very small. In the 70s the worsening of financial conditions of Italian firms induced entrepreneurs to ask for credit from the banks. In these years the multiple bank relationships emerged. On the one hand this implied that a firm received credit from more than one bank, on the other hand this implied that a good knowledge of the real economic conditions of firm was not easily available to banks. In many cases credit was provided also for personal trust reasons, without a deep investigation on the actual financial condition of the firm.

4.3 The data

In this section we analyze a subset of the AIDA database⁷. which provides information on 170.000 Italian industrial firms since 1992. It contains information on each firm characteristics (total net worth, total asset, solvency ratio, number of workers, added value). Moreover, for a subset of around 40000 firms, it is possible to identify which bank is financing each single firm, the period (short or long) of the loans, although not their amount. In 2003 (the year we investigated), the 40000 firms of our database were 37% micro (less than 10 workers), 45% small (between 10 and 50), 12% medium (workers between 50 and 250), 4% large (more than 250 employees). We complemented this dataset by using the Bank of Italy classification, which identifies the Italian banks in 5 groups (BI): larger banks (deposits > 45 billion Euros), large banks (between 20 and 45), medium banks (between 7 and 20), small banks (between 1 and 7), smaller banks (less than 1). We can thus identify the category of the banks for each individual firm

⁶A random graph is obtained starting from a set of N nodes and linking pairs of nodes randomly extracted from a uniform distribution among all the possible pairs of nodes (Dorogovtsev , 2003).

⁷The database AIDA is provided by Bureau van Dijk (url: www.bvdep.com)

and to analyze the behavior of banks of different sizes. Our sample is composed by 503 Italian banks and 5 foreign banks.⁸ The subset of Italian banks is composed by 9 larger, 8 large, 27 medium, 92 small, 357 smaller banks. In this work we focus on 75846 credit relationships (either short and long) of 39194 firms with 508 banks in the year 2003. In the following pictures, we use the following choice of colors:

- blue: largest banks
- green: large banks
- yellow: medium banks
- orange: small banks
- red: smaller banks

4.4 The network representation

We represent the banks-firms system as a network, using an approach based on the graph theory (De Masi and Gallegati, 2007 (b)).

In our case banks and firms constitute the nodes (Souma et al, 2003), while the links represent the credit relations among them. These kinds of networks, composed of two kinds of nodes, are called “bipartite networks”. In such a case, it is possible to extract two networks from the overall network, each one composed only of one kind of node: this method is called one-mode reduction and the two networks “projected networks”, i.e. they are obtained as a projection of the initial graph in the subspace composed by only one kind of node. From a mathematical point of view, a network is represented by an adjacency matrix. The element of the adjacency matrix a_{ij} indicates that a link exists between nodes i and j , that is $a_{ij} = 1$ if the bank i provides a loan to the firm j ; otherwise $a_{ij} = 0$. Let consider a set of firms $F = 1, 2, 3, 4$ (squares) and a set of banks $B = a, b, c, d, e, f$ (circles). The corresponding graph is given in the top panel of the Figure 1 .

This network is characterized by means of statistical measures. They can be divided into local measures (that represents the main properties of the node) and global measures (that represent the overall properties of the network).

The local measures characterise the properties of the single nodes. The most important ones are the degree (i.e. the number of links of the node), the clustering coefficient (i.e. the density of links near a node), the average degree of the neighbors, the centrality (i.e. a measure of the importance of a node in a network) (for a detailed description of all these measures see De Masi and Gallegati, 2007 (b)). We In the study of bipartite graph a very widely used approach is to separately study two networks that can be defined from the original network. If we call the two kinds of nodes as nodes A and B , we can study the network G_{A+B} which has the total set of nodes ($A + B$) or the networks G_A

⁸The universe of Italian banks in 2003 is composed by 11 larger, 11 large, 31 medium, 130 small, 605 smaller. Respect to the universe, our sample represents well the first three groups. It has the 71% of small banks and the 59% of smaller banks

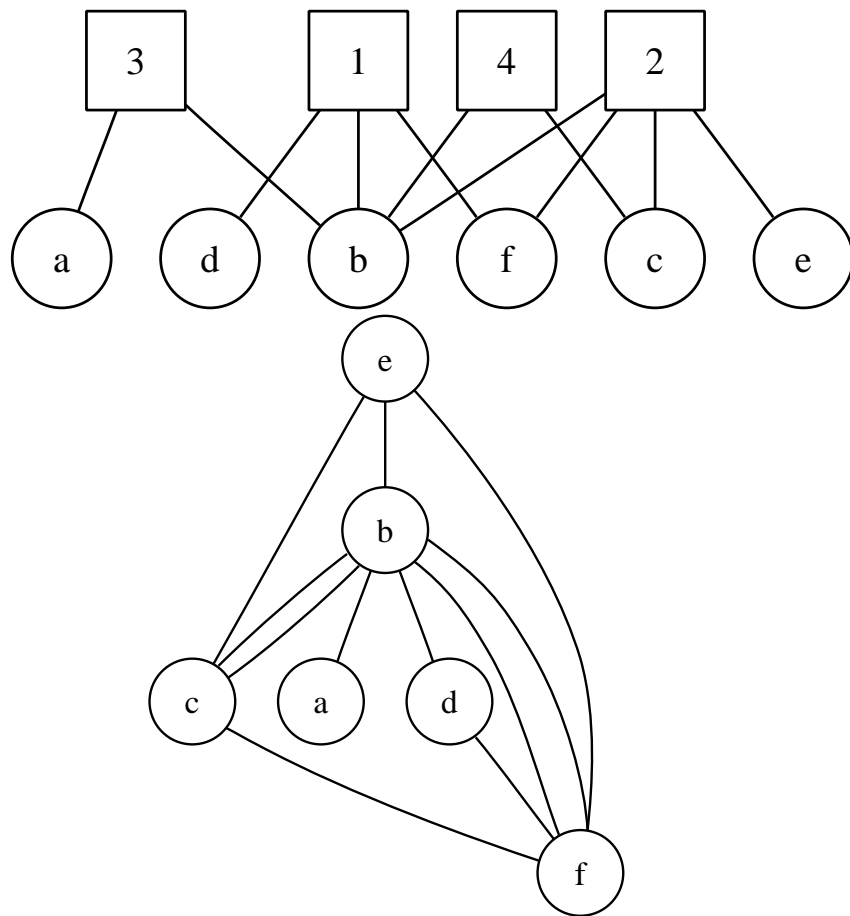


Figure 1: Bipartite graph and projected graph (one mode reduction on banks space). Firms are square and banks circles.

and G_B which have only nodes of kind A or B respectively (Strogatz, 2001; Newman, 2001).

In our case we can define the network of banks and the network of firms (De Masi G. and M. Gallegati, 2007 (a), De Masi G. and M. Gallegati, 2007 (b)). The first is the network of cofinancing banks: two banks are linked if they finance the same firm. The second is the network of co-financed firms, that is two firms are connected if they are financed by the same bank. Considering the same set of banks and firms of the first example, $F = 1, 2, 3, 4$ (squares) and $B = a, b, c, d, e, f$ (circles), the network projected into the subspace of banks (left panel) corresponding to the network of banks and firms is plotted in Fig.1.

Since the network of firms is highly disconnected, we focus on the bank network. This methodology allows us to identify common characteristics among banks linked to the same firm. The bank network is composed of 354 Italian banks and 3176 total undirected links. We have to define a weighted network, associated to a weighted adjacency matrix W : the weight associated to the link between two banks is the number of common firms they finance. Therefore, starting from the adjacency matrix of banks-firms contracts $a_{b,f}$, the general element of the matrix W is given by

$$w_{b,b'} = \sum_{f,f'} a_{b,f} \cdot a_{b',f'} \cdot \delta_{f,f'} \quad (1)$$

In Fig.2 the network of cofinancing banks is visualized, using the Kamada-Kaway algorithm⁹. From the plot of the network, we observe that the largest banks are in the core of the graph, and that the size of nodes decreases from the core to the leaves. The greatest Italian banks (red nodes) are the hubs of the network, i.e. they have most of the lending contracts, and they are surrounded by large and medium sized banks, while the smallest are at the very periphery of the network¹⁰.

We proceed with the analysis of the most important links in the projected network (De Masi G. and M. Gallegati, 2007 (a)). We focus on the connected subgraph. Instead of considering the whole weighted adjacency matrix W and the whole network we analyze a tree¹¹ with only $N - 1$ links. Many tree can be defined. This kind of trees are called spanning tree that is a subgraph which is a tree and connects all the vertices together. A single graph can have many different spanning trees. Considering the matrix W , we can define a distance matrix D , whose elements are:

$$d_{i,j} = \sqrt{2(1 - w_{i,j})} \quad (2)$$

Considering the weights $d_{i,j}$, we assign a weight to a spanning tree by computing the sum of the weights of the links in that spanning tree. A minimum spanning tree or minimum

⁹This algorithm, based on a mechanism of relaxation of a set of coupled harmonic oscillators, allows to visually detect the presence of groups of banks more connected to each other than to the rest of banks, since it puts the nodes of the same group close to each other in the 2-dimensional space

¹⁰A similar pattern has been observed for the Italian interbank market (Iori et al., 2007). The two behaviors are related to each other: in fact large banks, which maintain a big role in financing Italian firms use the Interbank market to obtain liquidity from small banks to give loans to the firms they finance.

¹¹A tree is a network without any cycle.

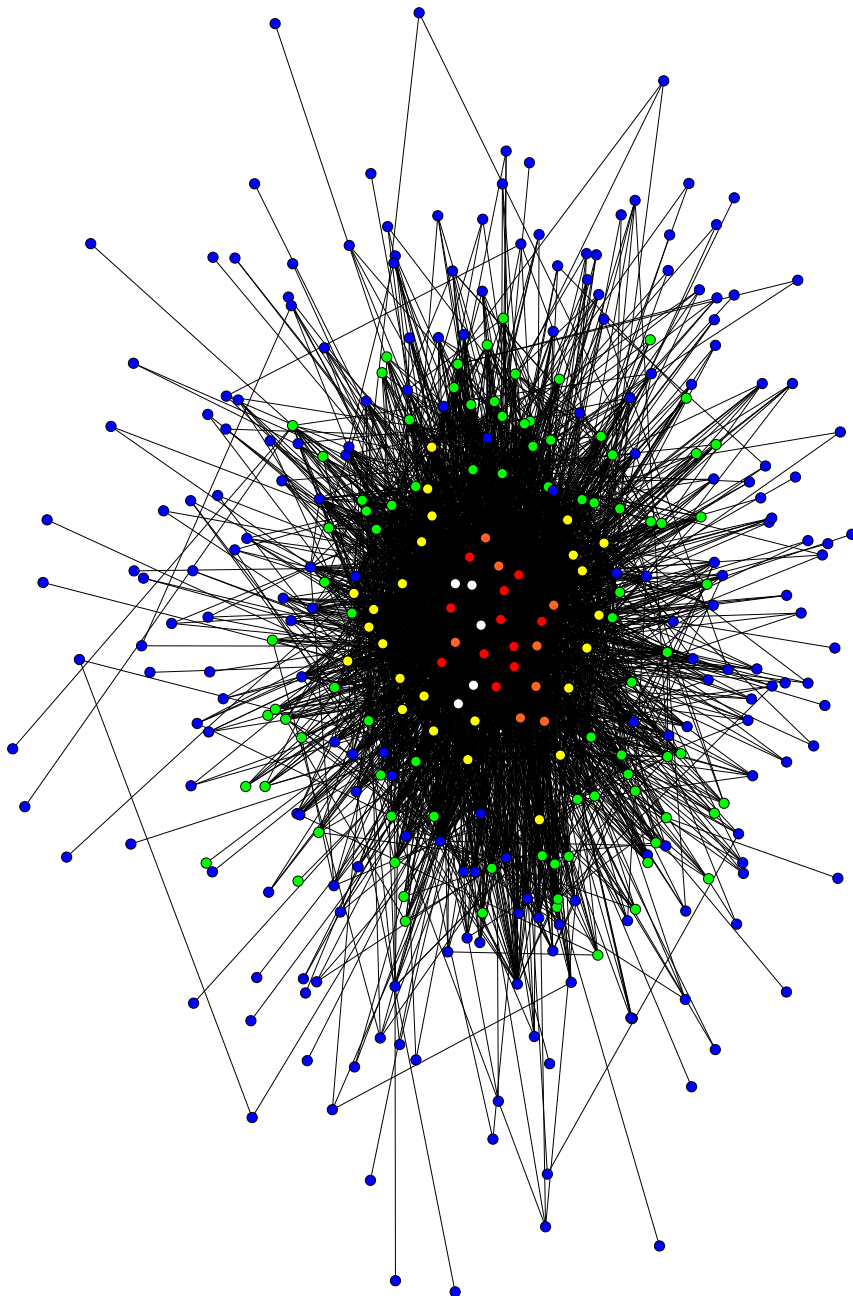


Figure 2: Graph representation of the one-mode reduction network on the subspace of banks. The convention for the use of colors is the same given above (the white dots are the foreign banks).

weight spanning tree is then a spanning tree with weight less than or equal to the weight of every other spanning tree, which select the most of information of matrix $d_{i,j}$. The algorithm used to construct the tree of banks is the Minimal Spanning Tree (MST) used also in the study of correlations among stocks in the financial market (Mantegna, 1999). The MST associated to our network is plotted in Fig. 3.

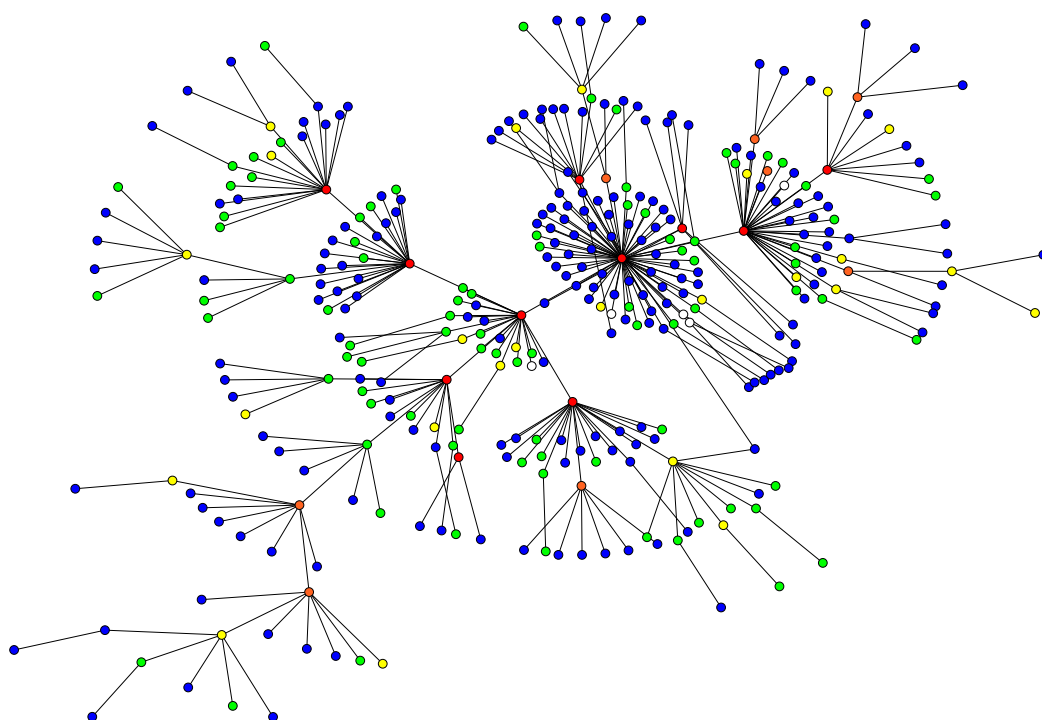


Figure 3: MST of Italian banks.

In the case of projected networks, the quantities under study are degree distribution, clustering coefficient, assortativity, betweenness centrality, diameter (Peltomaki et al, 2006). We have applied these tools to Italian banks-firms network (De Masi G. and M. Gallegati, 2007 (a), De Masi G. and M. Gallegati, 2007 (b)).

4.5 Main evidences

From the networks analysis we can study the correlation among the multilending and the intrinsic features of banks/firms.

1. First of all, we observe that the multilending is very widespread in Italy.

Hubs of network under study are big Italian banks. Studying the average degree of banks for each group, it emerges as expected that large banks have a greater

number of credit relationships but the number of links does not linearly grow with the size.

2. We calculated for each group of banks the average capital of firms financed by banks belonging to that group. Large banks finance mostly large firms, while small banks finance small firms. It can be emphasized that large firms receive credit from many large banks, while small firms are financed by small local banks and often by one bank alone.
3. It is not uncommon, anyway, that large firms with multiple linkages have relationships with banks of very different size. Firms with multiple linkages are big, often regarded as financially safe: because of it, small banks finance them, even without doing a rating investigation. Moreover large firms are usually able to provide hard information. On the contrary, small firms often find financing problems: therefore they are usually financed by small local banks, often better able to collect soft information than large banks.
4. From clustering measures it emerges that small banks (the ones with low degree) have a more pronounced tendency to finance a common subset of firms with other banks. This is not surprising because small banks are in general cooperative credit banks and local banks, which are usually quite sectorial (they finance small and local firms). On the other hand large banks finance firms from different sectors.
5. Moreover we studied the different tendency of multiple bank relationships of firms with different characteristics as asset, number of dependents, added value, solvency ratio. An analysis of the distribution of asset of firms with degree k , suggests that multiple relationships are preferred by large banks (because of the shift towards higher values of A), whereas single relationships are typical of small firms. A very different trend is observed considering the solvency ratio of firms. For solvency ratios lower than $s < 10\%$, we observe more firms with single lending than firms with multiple linkages; for $s > 10\%$, there are more firms with multiple lending. Healthy firms show a strong tendency to multiple lending, while firms with single linkage have in general low values of the solvency. This points out that single lending is more diffused for firms with low solvency ratio, i.e. riskier firms have single linkages and it stands for a greater difficulty of financially unsound firms to borrow from banks.
6. The algorithm of the MST allows to detect hierarchical clustering: in particular we find that the backbone of Italian banks system is composed by groups which corresponds to branches of the tree: in each branch we are able to identify the central node and the leaves. In particular we observe that central nodes are big banks (red and orange), while leaves are small banks (blue and green). Very often we can recognize regional clusters: branches are composed by banks of the same region. The hubs of the graph are the big Italian banks (red nodes): the most connected is BancaIntesa, followed by SanPaolo-Imi, Unicredit, BNL and MPS¹².

¹²Authors can provide detailed informations about the identities of nodes-banks.

The structure of the network is quite typical: around a big bank there are many smaller banks connected to the hub, while direct connections exist between large banks.

A comparison between the different network structure of bank-firm credit networks of many countries and the analysis of its evolution within a country will permit to understand how different institutional rules produce different emerging structures and business cycle phases. The object of this work was much narrow aiming to analyze the topology of the Italian banks-firms credit market, providing an empirical background to the *EURACE* implementation of credit market shown in the next section.

5 The Credit Market in EURACE

5.1 Introduction

The main actors in the Credit Market are Firms and Banks. Firms finance investments and production plans preferably with proper funds i.e retained earnings. When these funds are not sufficient, firms rely on external financing. Following the *Financial Policy Decision in EURACE* document, firms first apply for loans to the banks in the Credit Market. Firms could be credit-rationed. The decision is taken by the bank to which the firm applies and depends on the total amount of risk the bank is exposed to, as increased by the risk generated by the additional loan. If a firm is credit-rationed in the Credit Market it has other possibilities of financing, e.g. bonds and equity. Besides, households also apply for credit in order to finance their expenditure programs. Banks are profit seekers and their major role consists in financing the production activities of the firms: hence, when facing a potential client, banks must assess the convenience to serve it.

Banks active in most of industrialized Countries operate under the international Regulatory Regime Basel II. As recognized by Basel II rules, banks face three types of risk ¹³:

- the *credit risk*, i.e. the risk of losses due to a debtor's non-payment of a loan, which is related to the lending activity of the bank.
- the *operational risk*, which is the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events¹⁴.
- the *market risk*, which is the risk of losses due to market factors' movements¹⁵.

In EURACE we consider only the first type of risk, i.e. credit risk.

The Central Bank plays an important role. It helps banks by providing them with liquidity when they are in short supply. Moreover, the Central Bank has a role of monitoring the banking sector. In particular the Central Bank controls the level of risk

¹³Other types of risk are not considered fully quantifiable.

¹⁴The definition excludes systemic risk, legal risk and reputational risk.

¹⁵The main market risk factors are: equity risk, interest rate risk, currency risk and commodity risk.

the banking sector is facing. Furthermore, the Central Bank decides the lowest level of the interest rate, which is a reference value for the banking sector. Finally, the Central Bank provides the Government with funds by buying Government Bonds.

5.2 Credit supply

5.2.1 Deposits

For simplicity, we posit that each household has only one bank account. Households choose the bank where to deposit their savings by a preferential attachment mechanism, i.e. the probability that a bank is chosen is proportional to the number of its clients, or to its market share.

5.2.2 Loans

Here we present a simple mechanism where banks choose which loan request to serve, to which extent and at which price (the interest rate). The underlying principle is Basel II capital adequacy requirement compliance.

- Banks' capital is the cumulate of past profits.
- Banks may receive funds (in form of savings) from households. Banks pay no interest on this amount of money.
- Banks invest principally in loans to firms. They may consider portfolio diversification investing part of their liquid funds in Government bonds and private bonds and shares. Moreover, banks maintain part of the funds liquid as a kind of cautionary reserve. Bonds' and shares' risk contributes to the total risk which should remain below a determined level.
- If a bank is in short supply of liquidity (e.g. to face consumers' withdrawals), it may get credit from the Central Bank. Banks can not offer additional loans to the firms until debt to the Central Bank has not been extinguished.
- Banks deal with firms and households in a "first come, first served" basis. In principle, banks do not distinguish between firms and households, only risk matters.
- We denote the total actual risk of bank k as $R_k = \sum_j r_j$, where r_j is the risk related to client j , and with j belonging to the set of bank k 's clients Γ_k .
- The Basel II requirement we impose to be valid at every time is: $R_k \leq \alpha K$, with K being the capital of the bank. α is an exogenous policy parameter of the model, K evolves along time by earnings accumulation and R_k is the bank's control variable¹⁶.

¹⁶ α is an exogenous parameter for the banks, but it may be a control variable of the Central Bank in the wider model.

- If client j is a firm, the risk associated to it is given by the function $r_j = p_j(A_j, D_j, c_j)c_j$, with p_j being the probability of firm j not being able to repay its debt as assessed by the bank, A_j and D_j being respectively the assets and the total debt of the firm and c_j the amount of the loan. For instance, $p_j = 1 - \exp(-(D_j + c_j)/A_j)$. This mechanism ensures convenience for portfolio diversification since, ceteris paribus, the bank carries on a lower level of risk if it splits its credit into several loans instead of delivering the whole amount to a single big loan¹⁷. From this expression for p_j we expect from simulations the emergence of empirical results 1 and 3 in section (4.5).
- Households' risk depends on general economic conditions and is taken as given by the bank, since it is determined on the financial market.
- If credit demand d_j does not push R_k beyond Basel II threshold, the firm (or the household) j is not credit-rationed and gets the whole requested amount, such that $c_j = d_j$. Otherwise, the client is rationed and gets the maximum amount c_j^* of credit such that $R_k + c_j^* = \alpha K$ and $c_j^* < d_j$.
- The interest rate that a bank k asks to a client j applying for a loan d_j with risk r_j is $i_{k,j} = \bar{i} + \gamma_k r_j$ (alternatively, $i = \bar{i} + \gamma_k r_j^2$). \bar{i} is the official discount rate set by the Central Bank.
- γ_k , a bank-specific behavioral parameter, evolves over time depending on the past profits of the bank. We propose the following adaptive mechanism of adjustment:

$$\gamma_{k,t} = \gamma_{k,t-1} + \lambda(\gamma_{k,t-1} - \gamma_{k,t-2}) \frac{\pi_{t-1} - \pi_{t-2}}{\pi_{t-2}}$$

where $\lambda > 0$ is a sensitivity parameter¹⁸. The idea behind this mechanism is this: the bank repeats in each period what it did in the previous period if this strategy led to an increase in profits, otherwise it changes the strategy. So, for example, if an increase of γ in time $t - 1$ was followed by an increase in profits, then, in time t the bank will keep on increasing γ .

- Bank's profits in time t , π_t , read as follows:

$$\pi_t = \sum_j c_j i_j - \sum_j c_j \delta_j$$

where δ_j is the share of c_j that the firm j could not pay back. Thus, profits are simply interests minus bad debt.

- Firms are served one by one: a new customer must wait until the previous customer of the bank has been served (or denied service).

¹⁷The risk related to a single big loan is higher than the global risk related to several small loans given that the sum of these small loans equals the amount of the big loan.

¹⁸Theoretically there is the possibility that once $\gamma_{k,t-1}$ equals $\gamma_{k,t-2}$ then $\gamma_{k,t}$ will remain constant in time. We will investigate the case simulating the model.

5.3 Credit demand

Credit demand in EURACE is given by total loan requests coming from firms and households, the former to finance production and investments, the latter to relax budget constraints.

5.3.1 Households

The main households' role is to deposit money, but they also can ask for consumption credit if present labor income is not enough to satisfy their needs. The process regulating households' financing is the same as the firms'.

5.3.2 Firms

- Each firm sends n loan applications (n small). The basic choice process is random, but a number of preferential attachment mechanisms will be explored.
- Banks' offers are ranked by the proposed interest rate, the lower being the better.
- There is a multi-lending possibility for the firms. Each firm starts from the bank that offers the lowest interest rate and uses the whole amount this bank offers. Then, if the amount offered from the first bank was not enough, the firm goes to the second bank and so on, until credit request is fulfilled (if possible, otherwise the firm will be rationed). There are no further negotiations on the offered interest rates, even if the firm decided to take only part of the requested loan¹⁹.
- Debts are paid back within the next production cycle.
- Firms which are not able to pay back debts go bankrupt.

6 The entry & exit mechanism

Entry and exit mechanism in the consumer and investment goods markets is relatively simple, and only two requirements are imposed on it: at any time, both entry and exit should occur, and the turnover rate, i.e. entry plus exit rates, ought to be pro-cyclical. Entry and exit are two distinct processes, so the firm population can expand or shrink along time.

¹⁹This is a penalization to the firm since to a smaller loan would normally correspond a lower interest rate.

6.1 Entry

6.1.1 Firms

The number of new entrants at time $t + 1$, e , is computed by looking at the total profits realized in the industry at time t :

$$e_{t+1} = \eta \sum_j \pi_j(t),$$

with a suitable $\eta > 0$, which can evolve along time because of changing environmental characteristics. Alternatively, e can be a random draw e.g. from an exponential distribution with mean depending on aggregate profits. The industry profits can be computed either by including or by excluding bankrupted firms. Note that this mechanism simply resembles Marshall's concept of long-run competition, or Adam Smith's "invisible hand": profits are those that ultimately guide investment decisions. New entrants are random copies of a suitable subset of surviving incumbents. As an example, this subset can be defined as the lower half of the size distribution of incumbents (irrespective of profitability), to take into consideration the fact that new firms are generally smaller than those already in the market:

1. For each new entrant, a donor is randomly drawn from the donor subset;
2. All relevant entrant's variables are initialized with the values of the donor;
3. The entrant starts with no employees and no capital (or just a small amount). Production plans however are inherited from the donor firm. Hence, the appropriate number of new vacancies are immediately opened. Production in the first period of activity is entirely financed with bank loans.

This mechanism can be coupled with one regulating an on-the-run evolution of individual characteristics (parameters), by means, for example, of imitation of the best competitors.

A simpler alternative is that the subset of donors coincide with the whole set of survivors; this entry mechanism is sufficient to ensure a certain degree of evolutionism: firms with a higher fitness being more likely to survive, and consequently to be chosen as a donor. As a consequence, winning characteristics will spread in the population of firms.

6.1.2 Banks

Since the ultimate scope of the EURACE project is to develop a simulator for European economy, we build on the empirical evidence that Europe has experienced in the last decades a continuous process of concentration in the banking industry, with the total number of banks shrinking dramatically over time. Thus, our modeling strategy is to start at the beginning of each simulation run with a large number of small banks, which can in principle decrease along time: we expect then the emergence of a concentration path between banks, along with an increase in patrimonial soundness of surviving banks, leading to a stabilization of EURACE credit market.

6.2 Exit

6.2.1 Firms

As far as exit of firms is concerned, we simply have that:

1. Firms not able to repay their debt go bankrupt;
2. Workers employed in those firms become unemployed;
3. Physical capital of bankrupted firms gets lost;
4. There are no additional bankruptcy costs beside banks' bad debt.

The simplest solution is that these firms exit completely from the market, their workers become unemployed and physical capital is destroyed. Another more complicated solution could be that the bankrupted firms may be sold by a trustee of bankruptcy as a whole in the financial market or in parts (i.e. residual physical capital) to other firms.

6.2.2 Banks

In EURACE we also consider the possibility of banks' default: when bank capital falls below a certain level, the bank exits the market and is not replaced.

7 Simulations and basic Model Performance

At the moment, we want to stress that our goal is to show how agent-based models can be used in order to *construct* macroeconomics without recurring to Walrasian equilibrium concepts and to the representative agent tool. Hence, for the time being we will restrict just to a qualitative assessment of the simulation output produced by a simplified version of *EURACE* model in order to show its properties and potentialities. The model we have simulated here is simplified in all the markets: labor, good *and* credit: following our bottom-up modeling strategy, in future work we will expand the model step by step, incorporating in this basic framework firstly the EURACE credit market specification presented in section (5), and the capital goods sector. Then, we will simulate the EURACE model described in deliverable D7.1 incorporating the credit market of section (5).

7.1 The model

Consider a sequential world with only three markets: homogeneous labor, homogeneous consumption good and credit market. The sequence of events occurring in each period runs as follows:

1. Starting from the demand it expects to face, each operating firm determines the amount of output to be produced and the amount of labor to be hired. Expectations on future demand are updated adaptively.

2. A fully decentralized labor market opens. Firms then pay their wage bill W_{it} in order to start production.
3. If internal financial resources are in short supply for paying wages, firms fill in a fixed number of applications to obtain credit. Banks allocate credit collecting individual demands, sorting them in descending order according to the financial viability of firms, and satisfy them until all credit supply has been exhausted. The contractual interest rate is calculated applying a mark-up (function of financial viability) on an exogenously determined baseline.
4. Production takes the whole period t , regardless of the scale of output. At the beginning of t , firms pay their wage bill in order to start production. After production is completed, the market for goods opens. Firms post their offer price, while consumers are allowed to muddle through searching for a satisfying deal. If a firm ends up with excess supply, it gets rid of the unsold goods at zero costs. Individual productivity can be increased by an uncertain amount thanks to investments in R&D, determined as a random fraction, depending on financial fragility, of the last periods profits, π_{it-1} .
5. At the end of any t , firms collect revenues, calculate profits, update their net worth and, if internal resources are enough, pay back their debt obligations. Firms and banks check their financial viability as inherited from the past. They continue to operate if their net worth (a stock variable equal to the sum of past retained net profits) is positive; if, on the contrary, net worth is lower or equal to zero, they shut down due to bankruptcy. In the latter case, a string of new firms/banks equal in number to the bankrupted ones enter the market. Entrants are simply random copies of the smaller incumbents.

7.1.1 Production technology

Production is carried out by means of a constant return to scale technology, with labor L_{it} as the only input:

$$Y_{it} = \alpha_{it}L_{it} \quad \alpha_{it} > 0, \quad (3)$$

where α_{it} is labor productivity. The latter is assumed to evolve over time according to a first-order autoregressive stochastic process:

$$\alpha_{it+1} = \alpha_{it} + \omega_{it} \quad (4)$$

where ω_{it} is the realization of a random process exponentially distributed with mean $\mu_{it} = \sigma_{it}\pi_{it}/(P_{it}Q_{it})$: the higher is the amount of gross profits invested in R&D (scaled by nominal sales), the higher is the expected increase in productivity. An increase in R&D investment may be due either to larger profits and to a higher s , that is the fraction of gross profits devoted to investments aimed at increasing productivity. In simulations, s will be allowed to decrease with the financial fragility of the firm, computed as W_{it}/A_{it} . The law of motion of net worth, A_{it} , is given by:

$$A_{it} = A_{it-1} + (1 - \sigma_{it})(1 - \gamma)\pi_{it}, \quad (5)$$

where γ is the share of distributed profits.

If the net worth turns out to be negative, the firm becomes technically insolvent and goes bankrupt. As a consequence, it exits the market and it is replaced by a new firm.

7.1.2 The market for the consumption good

At the beginning of each time period, firms adjust adaptively either their price or their output level to take into account of changed business conditions. The choice between the two strategies depends on relative prices and unsold production of previous period; combining those signals, four mutually exclusive cases arise:

$$\left\{ \begin{array}{ll} D_{it}^e = Y_{it-1}(1 + \epsilon_{it}) & \text{se } S_{it-1} = 0 \wedge P_{it-1} > P_{t-1} \\ P_{it} = P_{it-1}(1 + \epsilon_{it}) & \text{se } S_{it-1} = 0 \wedge P_{it-1} \leq P_{t-1} \\ P_{it} = P_{it-1}(1 - \epsilon_{it}) & \text{se } S_{it-1} > 0 \wedge P_{it-1} > P_{t-1} \\ D_{it}^e = Y_{it-1}(1 - \epsilon_{it}) & \text{se } S_{it-1} > 0 \wedge P_{it-1} \leq P_{t-1} \end{array} \right. \quad (6)$$

where ϵ is a uniform random variable and D_{it}^e is the individual expected demand.

As regards the price adjustment mechanism, we assume that firms operate in a posted offer market. In spite of the good being homogeneous, asymmetric information and search costs imply that consumers may end up to buy from a firm regardless of its price not being the lowest. It follows that the law of one price does not necessarily apply. Prices are set considering both the unsold quantities during the last period (S_{it-1}), and the costs incurred in production.

If we let P_{it}^l be the lowest price at which firm i is able to cover its average costs:

$$P_{it}^l = \frac{W_{it}}{Y_{it}},$$

the price determined according to rule (6) is posted on price-tags if and only if $P_{it} \geq P_{it}^l$. In the opposite case, the firm posts the price P_{it}^l .

When the adjustment choice falls on quantities, the level of production planned at the beginning of period t (Y_{it}^d) depends on expected demand, $Y_{it}^d = D_{it}^e$. Thus, demand expectations are revised upward if a manager observes excess demand for its output, and downward when the opposite holds true.

Aggregate demand equals total wages paid by firms to workers employed in $t - 1$, as we assume that workers express individual demand functions with a marginal propensity to consumption equal to one. Given the lack of any market-clearing mechanism and that bargains on the good market are fully decentralized, consumers have to search for satisfying deals. The information acquisition technology is defined in terms of the number of firms Z a consumer can visit without incurring any cost. In other words, search costs are null as the consumer enters the market, continue to be null if he remained confined into his local market of size Z , but they become prohibitively high as soon as a consumer tries to search outside. In what follows, the identity of the Z firms associated to a generic consumer j are picked randomly at any time period t .

Consumers enter the market sequentially, the picking order being determined randomly at any time period t . Each purchaser j is allowed to visit Z firms to detect the price

posted by each one of them. Prices (and the corresponding firms) are then sorted in ascending order, from the lowest to the highest. Consumer j tries to spend all the income gained during the last period in goods of the cheapest firm in his local market. If the cheapest firm has not enough available output to satisfy j 's needs, the latter tries to spend his remaining income buying from the firm with the second lowest price, and so on. If j does not succeed in spending his whole income after he visited Z firms, he saves what remain for the following period. For the sake of simplicity, the interest rate is assumed to be equal to 0.

After the market for consumption goods has closed, the i th firm has made sales for Q_{it} , at the price P_{it} . Accordingly, i 's revenues are $R_{it} = P_{it}Q_{it}$. Due to the decentralized buying-selling process among firms and consumers it is possible that a firm remains with unsold quantities ($S_{it} > 0$). In the following period, the variable S will be considered in adjusting firms' prices or quantities, as explained above.

7.1.3 The labor market

Firms set their labor demand L_{it}^d on the basis of their desired level of production. From equation (3), it follows that the number of job openings set by firm i at time t is simply given by:

$$L_{it}^d = \frac{Y_{it}^d}{\alpha_{it}}.$$

We assume that workers supply inelastically one unit of labor per period. Each worker sends M applications to as many firms: the first one to the firm in which he worked in the previous period (if employed), and $M - 1$ at random (M if unemployed in $t - 1$). Workers are therefore characterized by a sort of loyalty to their last employer, on the one hand, and by a desire to insure themselves against the risk of unemployment by diversifying in a portfolio of hiring opportunities, on the other one.

Firm i may face two alternative situations: a) $L_{it}^d \leq L_{it-1}$, that is the desired labor demand at time t is lower than the number of people employed during the previous period; b) $L_{it}^d > L_{it-1}$, that is firm i wants to increase its workforce. In this case, i keeps all its past employees and looks for $L_{it}^d - L_{it-1}$ new workers.

Decentralized labor markets (i.e., one for each firm) are closed sequentially according to an order randomly chosen at each time step.

If hired, a worker sign a contract which determines the wage level for 8 periods. The wage offered to him by firm i in period t is calculated according to the following rule:

$$w_{it}^b = \begin{cases} w_{it-1}^b(1 + \eta_{it}) & \text{if } L_{it-1} < L_{it}^d \\ w_{it-1}^b & \text{if } L_{it-1} \geq L_{it}^d \end{cases} \quad (7)$$

where η_{it} is an idiosyncratic shock uniformly distributed on the interval (0,1).

Unemployed workers will accept the proposal from the firm which pays the higher wage.

7.1.4 The credit market

If net worth is insufficient to run production, firms can try to obtain additional funds by recurring to a credit market. Credit demand is simply given by:

$$B_{it} = W_{it} - A_{it} \quad (8)$$

Each bank offers to the generic firm i a standard single-period debt contract, which consists of an interest rate offer $r_{ni}(l)$ - where l is i 's leverage ratio.

The contractual interest rate offered by bank n to firm i is determined as a mark-up over a discount rate set by a central monetary authority, which for simplicity we assume to be constant:

$$r = R * (1 + \phi f(l))$$

The mark-up f the bank charges over the official discount rate reflects a risk premium which increases with the financial fragility of the borrower. Finally, the stochastic shock ϕ captures random variations in banks' operating costs.

A firm which needs external finance to implement its desired production program can explore the bank loan market by randomly picking H , with $0 < H < N$, trials. Such a firm then sorts the offers it receives in ascending order, and chooses the bank ranked first.

Credit supply is just a multiple of bank equity base.

7.2 Results

Figure (4) shows long-run performance of our simple model, which is able to display highly realistic patterns in main economic variables. What neatly emerges is the capacity by the economic system to self-organize along time in increasing stable corridors, just on the basis of dispersed, individual, out-of-equilibrium transactions. No general equilibrium condition has been imposed, nevertheless the system reaches a spontaneous, although turbulent, order.

Below we can see comovements²⁰ between selected variables and output produced by the model, and the corresponding real values for U.S. economy. Not only signs and statistical significance, but also absolute values are strikingly similar, showing that even a very simple version of EURACE model is able to capture stylized facts with no *ad hoc* assumption, and is able to challenge DSGE theory on its own field.

7.3 The two sector Model

In the second model (not simulated yet) we consider a sequential economy populated by a large number of firms F , a large number of Household (which are made of workers/consumers) H and Banks (B). We now distinguish two kinds of firms operating within two distinct industries: a downstream industry made of firms (F^d) producing a homogeneous Consumption Good (CG) and an upstream industry which performs R&D and whose firms (F^u) produce heterogeneous Intermediate (or Investment) Goods (IG).

²⁰Cross-correlations between cyclical components extracted by means of Hodrick-Prescott filter.

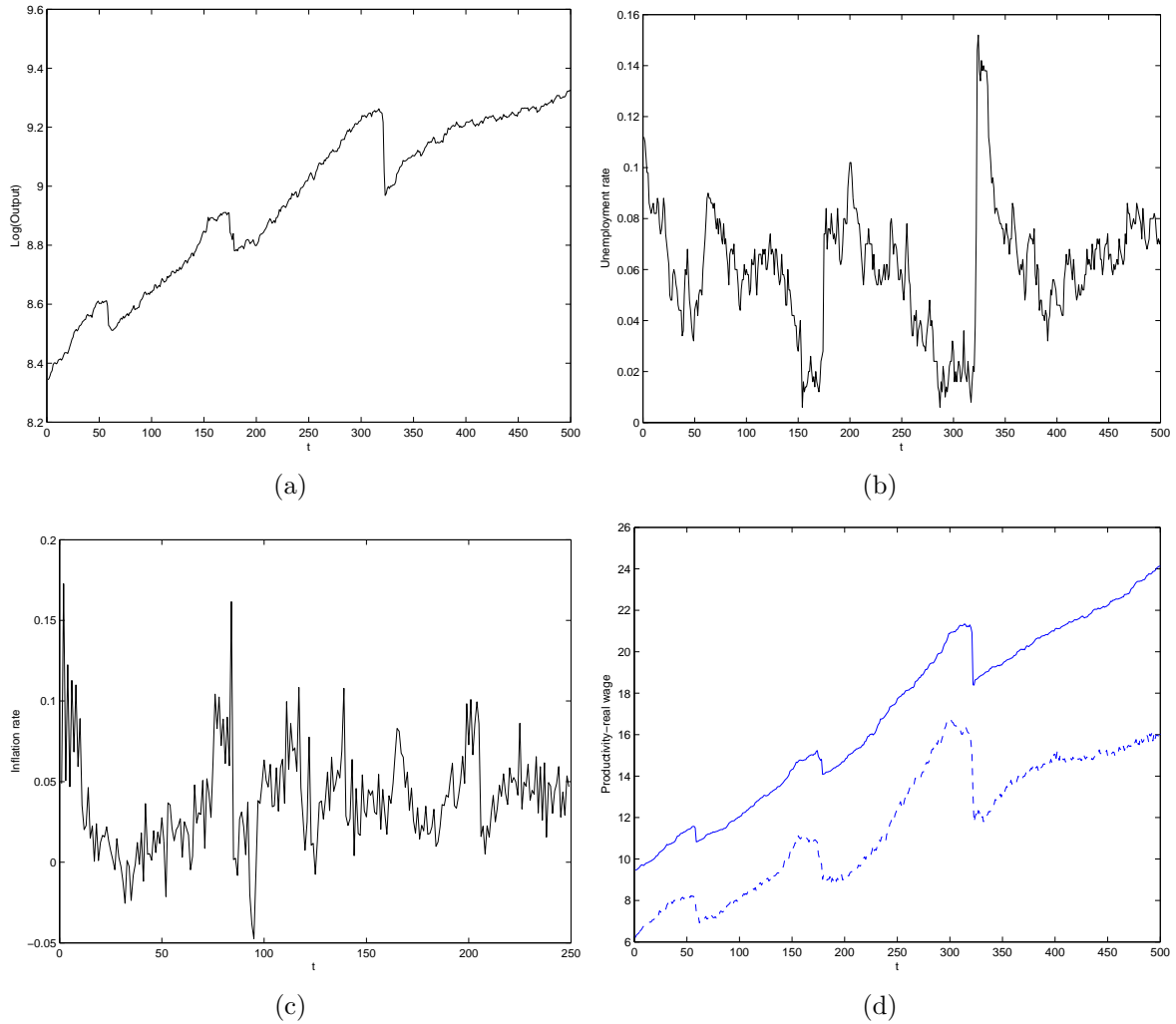


Figure 4: Simulation macroeconomics regularities: (a) GDP (in log); (b) unemployment rate; (c) inflation rate; (d) productivity-real wage.

All the agents undertake decisions at discrete times $t = 1, \dots, T$ on the markets for the Investment and Consumption Goods and Labor and Credit services.

Each firm is run by a single manager. All of them share the following characteristics.

Firstly, they use decision rules with bounded rationality. This means that they choose prices and quantities in an adaptive way by looking at their most recent past ²¹.

Secondly, managers try to finance their operating costs at first from internal funds. If these funds do not cover the whole costs, firms ask for loans to the Banks in the Credit Market.

Three kinds of markets are involved: a Labor Market (LM), a Goods Market (GM) and a Credit Market (CM). The model reproduces a *verticalized* economy since the F^u firms

²¹we will consider learning processes and other ways of decision making processes in a more sophisticated version of the model

	-4	-3	-2	-1	0	1	2	3	4
Employment									
USA	-0.101	0.096	0.345	0.611	0.803	0.844	0.728	0.549	0.339
Model	0.127	0.226	0.344	0.478	0.623	0.544	0.408	0.280	0.172
Productivity									
USA	0.136	0.376	0.629	0.815	0.926	0.676	0.401	0.111	-0.112
Model	0.239	0.387	0.577	0.779	0.883	0.694	0.490	0.320	0.178
Price index									
USA	-0.525	-0.526	-0.471	-0.375	-0.273	-0.132	-0.011	0.070	0.118
Model	-0.231	-0.255	-0.266	-0.256	-0.188	-0.093	-0.028	0.004	0.030
Interest rate									
USA	-0.151	-0.107	-0.082	-0.028	0.038	0.066	0.080	0.097	0.115
Model	0.020	0.019	0.003	0.000	-0.013	-0.006	0.026	0.002	0.000
Real wage									
USA	0.608	0.662	0.676	0.630	0.511	0.350	0.166	-0.019	-0.190
Model	0.19	0.353	0.546	0.734	0.842	0.729	0.546	0.377	0.235

Table 1: Cross-correlations with output.

produce IG and invest in R&D, while F^d firms buy technological innovations on the intermediate good market (IGM) to produce their output for the consumption good market (CGM).

All the markets are characterized by decentralized search and matching processes. Interaction and adjustment involve dynamics at the individual level. Macroscopic regularities emerge from the interactions of the agents at the microscopic level. Due to the absence of market-clearing mechanisms, the economy may be characterized by the contemporaneous presence of persistent involuntary unemployment, unsold production and excess of individual demand.

We suppose that firms in the downstream industry (F^d) produce the CG using technological innovations (IG) from the upstream industry firms (F^u) and low skilled workers. On the contrary, F^u firms produce IG using high skilled workers²². Firms and Workers interact in the Labor Market and the previous assumption requires this market to be split into two sub-markets: a high skilled workers' labor market (LM^{hs}) and a low skilled workers' labor market (LM^{ls}).

To complete the scheme, we notice that Households (H) interact with Firms (F) in the LM supplying labor to earn a wage/salary. Even though we have two kind of workers (high skilled engineers and low skilled workers), in the LM^{hs} and LM^{ls} sub-markets the matching mechanism will be the same.

The wage is assumed to be completely spent in consumption²³. If the consumer does not succeed in spending the whole income, he saves what remains for the following

²²Aghion and Howitt (1992) present a model where skills of workers do matter for the production of IG, but not for the production of CG

²³It may be integrated with financial instruments in CM in the hand of Banks, like assets linked to the financial performance of Firms, but our model does not provide any financial market. Nevertheless, a Financial Market Model is being developed within the framework of the EURACE Project (WP6)

period. To keep it simple, there is no interest gain on this amount of money. In order to run the planned production, upstream and downstream firms could be financially constrained. The financial liquidity of firms may be insufficient to pay in advance the whole wage bill and the R &D or the Investments of firms. If this is the case, firms may turn to the credit market that operates in our economy providing liquidity. Credit demand of each firm is equal to labor and investment costs (in R&D or new machines) excess over its financial resources (or reserves). Firms borrow at a fixed interest rate, $r_{i,t}$ at the beginning of the period. The payment is carried out entirely at the end of the period in a unique installment. Firms that do not manage to pay back their debt go bankrupt. The lending bank suffers a bad debt equal to the difference between lent money and the part of the debt paid back. The matching mechanism in the Credit Market is slightly different from the one presented in the previous model. Firms may face credit rationing and this decision, taken by the banks, depends on an indicator of firms' financial health. Finally, in a third transitory model the EURACE credit market specification shown in section (5) will be implemented in the simplified model.

8 Appendix: Stylized Facts

In what follows, we sketch a brief guide of stylized facts, usable as first benchmark in order to validate and/or calibrate future EURACE model. The list is obviously partial and it does not make any kind of ranking among the facts.

8.1 Industrial dynamics facts²⁴

1. Firms' size distribution is right skew and it is described, at least on the right tail, by a power law (Axtell, 2001; Gaffeo *et al.*, 2003; Ramsden, Kiss-Haypal, 2000; Okuyama *et al.*, 1999);
2. Firms' growth rate are Laplace distributed (Stanley *et al.*, 1996; Bottazzi, Secchi, 2003);
3. Given the firm's size, the average growth rate is decreasing with its age;
4. The average growth rate of surviving firms decreases as firms' size increases;
5. The probability of surviving is positively correlated with firm's size and age;
6. Probabilities of entry and exit decrease with firms' size;
7. Number of firms firstly increases to a maximum and then tends to remain stable at a lower level (Jovanovic and MacDonald, 1994);
8. There is a high correlation between entry and exit rates (Geroski, 1991);

²⁴Sutton, 1997; Caves, 1998; Geroski, 1995.

8.2 Financial stylized facts

1. The rate of interest is *a-* or moderately *pro*-cyclical (Gallegati, Gallegati, 1996);
2. There is no correlation between the rate of interest and debt (Delli Gatti *et al.*, 2004);
3. The ratio between firms' and banks' capital is approximately constant (Gallegati *et al.*, 2003);
4. The distribution of profits and of the amount of loans is power law (Fujiwara, 2003);
5. The distribution of *bad debts* is Weibull distributed (Delli Gatti *et al.*, 2005);
6. A higher equity ratio is associated with lower volatility of profits (Delli Gatti *et al.*, 2004);
7. The rate of return of capital and the equity ratio are positively correlated (Delli Gatti *et al.*, 2004);
8. Financial ratios are good predictors of firms failure and the equity ratio deteriorates almost monotonically as the date of bankruptcy approaches (Beaver, 1966);

8.3 Business cycle facts

1. The distribution of exits by age is exponential (Steindl, 1965; Fujiwara, 2003);
2. The duration of recession phases is power law, while expansions durations is exponential (Ausloos *et al.*, 2004);
3. The GDP autocorrelation is near to unity (Gallegati, Stanca, 1999);
4. Firm size distribution shifts over the business cycle: during expansions it becomes less steep (Gaffeo *et al.*, 2003b);
5. Wages are at best weakly procyclical (Greenwald, Stiglitz, 1988);
6. Hours and employment variations are strongly procyclical (Ibid.);
7. There is a positive relationship between output and productivity growth (Ibid.);
8. Productivity is strongly procyclical (Ibid.);
9. GDP cycle shows positive serial correlation (Ibid.);
10. Good market nominal price cycle presents high persistency (Ibid.);
11. Producers' durable equipment investment fluctuates less than aggregate investments (Ibid.);

12. Inventories fluctuates more than aggregate investments (Ibid.);
13. Bankruptcies are power law distributed (Di guilmi *et al.*, 2004b).

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