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Computational experiments of policy design on Goods, Labour and Credit Markets

Responsible authors:
Fabio Clementi, Saul Desiderio, Mauro Gallegati, Umberto Gostoli,
Alberto Russo, Gabriele Tedeschi
Universita' Politecnica delle Marche (Ancona)

Contributions from:
University of Bielefeld
University of Sheffield

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Contents

Abstract

This document contains the modeling specifications for the EURACE Goods, Labour and Credit Markets.

We will focus our attention on the implementation of the Credit Market model. A C and a $FLAME$ versions are developed. The goal of these codes is, on one hand, to test the economic variables of the Credit Market, on the other, to prepare our market to the complete integration with the EURACE models. Moreover, a preliminar set of computational experiments has been performed.

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1 Introduction

This document contains the modeling specifications and implementation for the credit market in the *EURACE* project, which developes an agent-based platform for European economic policy design, applying a bottom-up approach in order to build a micro-founded model of the European economy. This model will allow to test the effect of macroeconomic policy in an economic system composed by a large number of interacting heterogenous agents.
All the agents (firms, households, etc.) may be involved in different economic activities (*i.e.* markets) and are characterized by different actions or *functions*. These actions are grouped in distinct *roles*. Roles are ideal interfaces between agents and markets. For example, firms selling goods are playing a role in the 'consumption goods market' using, so, a set of functions related to that role. Instead, when they are asking loans, they are entering the 'credit market' role using the set of functions this role requires.

The complete $EURACE$ model is made up of eigth software modules. Each module is the *FLAME* implementation of a particular economic role for different agents. The list of modules is:

1. Artificial Labor Market;
2. Artificial Consumption goods Market;
3. Artificial Investment goods Market;
4. Artificial Credit Market;
5. Artificial Financial Market;

6. Firm's financial management role;
7. Government;
8. Eurostat.

Each module implements a set of functions for all those agents playing a certain role. So, for example, the 'Credit Market' module implements functions for banks and firms' 'credit market' role. The last two modules (7-8) deal with roles played by one single agent respectively.

Since roles are interlinked following the logical structure of *EURACE* model, the modules have to be integrated accordingly. The integration procedure of all the modules is threefold. In a first step, an isolated version of each module is implemented and tested as a stand-alone program, whose interconnections with the other modules are substituted by random numbers or by dummy modules[1]. Computational experiments are then performed with it. In a second step, a partial integration is given by pairwise combining each isolated model. This intermediate step permits to test the interfaces among modules. Finally, all the modules are integrated according to the complete picture of *EURACE* model.

In this paper we will describe in detail the theoretical model for the credit market and the implementation in C language and in *FLAME* (ACM), along with the results of some computational experiment.

Whereas the *C* version of Credit Market is a stand-alone model using random variables to simulate the other modules (first step of integration procedure) and is not integrable with them, the *FLAME* version integrates a dummy 'Firm's Financial Management Role' module and a multi-purpose 'Dummy Agent' with our Credit Market, which is then ready to be integrated with the other real modules of the complete *EURACE* model. The structure of our ACM is displayed in figure (6).

Consumption goods and labor markets are treated in detail in Bielefeld Unit deliverable D7.2.

We proceed in the following way. In the next section we give the mathematical description of the Credit Market. In section 3 we present simulations and results of the implementation in *C* language. Here the goal is to estimate the weight of the demand of credit in the system. Finally, in section 4 the model is implemented using *FLAME*. The *FLAME* implementation could be considered as a preliminary step to achieve the complete integration among the EURACE models. In appendix, we report some stylized facts related to the size distribution of banks, usable as benchmark in order to validate and/or calibrate the integrated EURACE model.

2 Mathematical description of Credit Market

The main actors in the Credit Market are Firms and Banks. Firms finance investments and production plans preferably with proper funds i.e retained earnings. When these

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[1] A dummy module is a piece of software sharing the same architectural design with its real counterpart, but whose internal processes are replaced by stochastic processes.

funds are not sufficient, firms rely on external financing. Following the Financial Policy Decision in EURACE document, firms first apply for loans to the banks in the Credit Market. Firms could be creditrationed. The decision is taken by the bank to which the firm applies and depends on the total amount of risk the bank is exposed to, as increased by the risk generated by the additional loan. If a firm is credit-rationed in the Credit Market it has other possibilities of financing, e.g. bonds and equity.
Banks are profit seekers. Their role consists in financing the production activities of the firms. Banks operate under a Regulatory Regime called Basel II. As recognized by Basel II rules, banks face three types of risk[2]:

- the credit risk, i.e. the risk of losses due to a debtor's non-payment of a loan, which is related to the lending activity of the bank.
- the operational risk, which is the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events[3].
- the market risk, which is the risk of losses due to market factors' movements[4].

We consider here only the first type of risk, i.e. credit risk.
The Central Bank plays an important role. It helps banks by providing them with liquidity when they are in short supply. Moreover, the Central Bank has a role of monitoring the banking sector. In particular the Central Bank controls the level of risk the banking sector is facing. Furthermore, the Central Bank decides the lowest level of the interest rate, which is a reference value for the banking sector. Finally, the Central Bank provides the Government with funds by buying Government Bonds.

2.1 The model

The model is a network with F firms and B banks.
The primary purpose of banks is to channel funds received from deposits towards loans to firms.
The amount of credit requested by the firms is an exogenous input in this model.
A loan made at time t to the firm f has to be payed back by $t{+}\tau$ periods. Each period, the firm has to pay a fixed share of its debt plus the interests on debt.
Each bank receives stochastic shocks to its supply of liquidity, S_t^b, arising ultimately from the deposit of customers. Since deposits are unpredictable and exogenous to the model, a bank may find itself unable to give credit due to the illiquidity, that is its lack of deposits. We interpret these shocks broadly to include both cash deposits and withdrawals.
Instead of strictly controlling aggregate deposits, we assume they are idiosyncratic random variables uniformly distributed.
The system operates in a discrete time, which is denoted by $t=0,1,2,\ldots,T$.
Credit linkages between banks b and firms f are defined by connectivity matrix, $M_{b,f}$.

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[2] Other types of risk are not considered fully quantifiable

[3] The definition excludes systemic risk, legal risk and reputational risk.

[4] The main market risk factors are: equity risk, interest rate risk, currency risk and commodity risk.

If the firm f links with the bank b, $M_{b,f}$ is equal to one, otherwise $M_{b,f}$ is equal to zero. When firms need loan, a connectivity matrix is randomly chosen.

Usually in the networks which define a local interaction one assumes that each node interacts directly with only a finite number of others in the population. So to reproduce incomplete information each firm links with maximum three banks chosen in a random way; it means that a firm knows the credit conditions, such as the interest rate, of three banks at the most.

To know the credit condition of banks the firm links with, it has to reveal its assets A_t^f, its total debt D_t^f, that is the sum of the loan that the firm has received from each bank and not payed back yet, and its credit request c_t^f, calculating at the time t. Using these variables, the banks calculate the credit risk r_t^f associated to that firm, according to:

$$r_t^f = \hat{p}_t^f \frac{c_t^f}{A_t^f} \tag{1}$$

where A_t^f being the assets that, like the demand of credit, is an idiosyncratic random variable and $\hat{p}_t^f$ being the probability of the firm f not being able to repay its debt,

$$\hat{p}_t^f = 1 - e^{-(\frac{D_t^f + c_t^f}{A_t^f})} \tag{2}$$

Using this data each contacted bank offers its interest rate, $i_t^{b,f}$, to the firm f applying for a loan c_t^f with risk r_t^f:

$$i_t^{b,f} = \hat{\imath} + \gamma_t^b r_t^f \tag{3}$$

where $\hat{\imath}$ is set by the Central Bank and γ_t^b is a bank specific variable evolving over time and depending on the past profits of the bank.

We propose the following mechanism of adjustment:

$$\gamma_t^b = \gamma_{t-1}^b + \lambda(\gamma_{t-1}^b - \gamma_{t-2}^b)\frac{\pi_{t-1}^b - \pi_{t-2}^b}{\pi_{t-2}^b} + \epsilon_t \tag{4}$$

where $\lambda > 0$ is a sensitivity parameter, π^b is the bank's profit and ϵ_t is a noise inducted component uniformly distributed in the interval $[0, 0.01]$.[5]

After this first consulting meeting, each firm asks the banks it links with for credit starting with the one with the lowest interest rate. Banks deal with firms in a "first come, first served" basis.

If the total risk of the bank, R_t^b, is lower than a percentage α of its capital K_t^b, where K_t^b is the cumulate of past profits and $R_t^b = \sum_f^N r_t^f$, with r_t^f equal to eq. (1) and N number of firms f the bank links with, and the bank has enough supply of liquidity, S_t^b, then the bank grants the loan; if $R_t^b < \alpha K_t^b$ and the bank has not enough supply of liquidity, then the bank grants the loan asking money to the central bank. Banks can not offer additional loans to the firms until debt to the Central Bank has not been extinguished (each bank can ask money to CB just one time, then before asking again it has to pay its previous debt); if $R_t^b \geq \alpha K_t^b$ the firm asks the bank with the second lowest interest rate.

To sum up, therefore, banks loan (do not loan) when:

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[5] ϵ is introduced because, there is the possibility that once γ_{t-1}^b equals γ_{t-2}^b then γ_t^b will remain constant in time.

- $R^b_t < \alpha K^b_t$ and $S^b_t > 0$ then $c^f_t = d^f_t$
- $R^b_t < \alpha K^b_t$ and $S^b_t <= 0$ there are two sub-cases:
    - if $CB = 0$ then $c^f_t = d^f_t$
    - if $CB > 0$ then $d^f_t = 0$
- $R^b_t \geq \alpha K^b_t$ then $d^f_t = 0$

with c^f_t and d^f_t, respectively, requested and obtained credit from firm *f*.
$R^b_t = \alpha K^b_t$ has to be interpreted as the Basel II threshold. This threshold may be viewed as an helpful tool to reduce banks risk, in particular the credit risk. So, $c^f_t = d^f_t$ if this does not push R above the Basel II threshold, otherwise we have credit rationing.
The bank supply of liquidity, S^b_{t+1}, evolves according to:

$$S^b_{t+1} = V^b_t + \sum_{f=1}^{N} \frac{1}{\Delta} d^f_t (i^{b,f}_t + 1) - \sum_{f=1}^{N} \omega^f_t d^f_t - \sum_{f=1}^{N} d^f_t - CB^b_t \tag{5}$$

where the first term, V^b_t, denotes deposits held by the general public in bank b, the second term is the fixed share[6], $\frac{1}{\Delta}$, of debt plus the interests on debt the bank receives from the N firms it loans to[7], the third term is the share, ω, of loan that firm could not pay back[8], the fourth term shows total loan of bank b in t and the last term, CB^b_t, denotes borrowing by bank b from the Central Bank. Note that CB^b can be positive or zero. If borrowing is positive, that is $CB^b > 0$, as soon as the bank supply of liquidity is enough, the bank has to pay the Central Bank back. As long as borrowing is not zero again, the bank can not loan to the firms. Banks do not pay interest to the CB. At $t = 0$, the value of $S^b_{t=0}$ is chosen exogenously.
Banks are profit seekers. A bank's profits in time t read as follows:

$$\pi^b_t = \sum_{f=1}^{N} \frac{1}{\Delta} i^{b,f}_t d^f_t - \sum_{f=1}^{N} \omega^f_t d^f_t \tag{6}$$

As described above, the behaviour of firms is not investigate in this model. The amount of credit requested by the firm *f*, c^f, and its assets, A^f, are exogenous inputs in this model.
Each period firms make profits/losses, π^f_t, which are random variables uniformly distributed in the interval $\in [-Min, +Max]$. Firms' assets, A^f, are the cumulate of profits/losses, π_t.
If the firm's assets, at the time t, are enough to pay back the fixed share of debt plus the interests on debt borrowed from each bank b at the time *t-1*, the firm pays its debts back, otherwise goes bankrupt. The assets of the failure firms are proportionally subdivided among creditors.
When a firm goes bankrupt a new one enters into the credit market asking for loan.

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[6] Δ is the fixed time by which firms have to pay their debt back.

[7] If, for instance, the firm f has a debt of 10, an interest rate of 2% and $\Delta = 10$, it means each period it has to pay back 1+0.02 for ten periods; after 10 period the debt is repaid.

[8] We will investigate the reasons of firms failure later in the paper.

3 C implementation

In this section we analyze, via numerical simulations, various properties of the Credit Market. In the simulations we considered (in succession) the C and $FLAME$ version. We chose the number of banks $B = 100$ and firms $F = 10000$.
As described above, the behaviour of firms and households is not investigated in the model. So the amount of credit requested by the firm f, c^f, its profits, π_t, and the deposits of customers, V_t, are exogenous inputs in this model.
Each period firms make profits/losses, π_t^f, which are random variables uniformly distributed in the interval $[-Min, +Max]$ and have a random demand of loan uniformly distributed in the interval $[0, L_{max}]$. Firms' assets, A^f, are the cumulate of profits/losses, π_t.
Households deposit and withdraw. The aggregate deposits, V_t, are idiosyncratic random variables uniformly distributed in the interval $[-Min, +Max]$.
In the following simulations we choose $L_{max} = 1000$, multiply by ζ, and $[-Min, +Max] = \pm 100000$.
As we will show the parameter ζ is fundamental to understand what happens in the system when the amount of credit increases, ceteris paribus.
We also fix $\Delta = 10$, - it means that firms have to pay their debt back by 10 periods, according to equations 5 - the interest rate of the Central Bank, $\hat{i} = 2\%$, the λ of the equation 4 is equal to 0.2 and the α of the Basel II threshold is equal to 0.02.

3.1 Results

The results reported here are the outcome of a simulation of 1000 steps. The results of the models are simulated numerically. We center the analysis on the dynamic of some endogenous variables of the model by varying the parameter ζ, which drives the amount of credit request by firms. What we want to investigate is the reaction of the Credit Market in front of big jumps of the demand of loan, ceteris paribus. It is a good exercise to understand in advance the response of our market to exogenous shocks, that is, all those shocks which will be generated by the integration of the EURACE model.
The figure. (1) displays a sample path of the logarithm of the average interest rate for two different value of ζ.
As it is obvious the lowest interest rate is equal to Central Bank interest rate, set at 2%. What is clear is that, increasing the demand of credit and not modifying the supply, interest rates increase neatly. We will give an economic interpretation of this phenomenon later.
The graph (2) shows for the same value of ζ the average bad debt, that is the quote of debt that firms are not able to pay back to banks.
The red line is for a $\zeta = 50$ and the black for a $\zeta = 1$. Also in this case the higher is ζ the higher is the bad debt.
The economic reason of the correlated movement of interest rates and bad debt has been showed by Stigliz and Greenwald (2003). The central determinant of the level of economic activity is the supply of credit, which is generated by deposits and debt payments, according to equation 5. When $\zeta = 1$ there is a correlation between demand

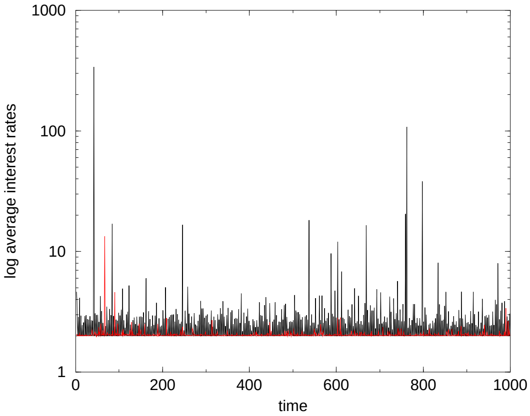

Figure 1: Log average interest rate for $\zeta = 1$ (red line) and for $\zeta = 50$ (black line).

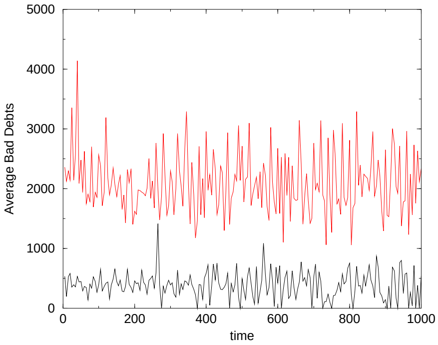

Figure 2: Average bad debt for $\zeta = 1$ (black line) and $\zeta = 50$ (red line).

and supply, increasing ζ and leaving deposits to be constant we lose this correlation. Whenever an economic exchange without an immediate flow of money occurs, a credit-debt relationship emerges. Failure of fulfilling debt commitments could lead to bankruptcy chains. If debt commitments are not fulfilled, bad debt increases with the likely consequence of an increase in the interest rate. This latter leads to more bankruptcies as showed by the picture (3), where is displayed the firms default rates for the two different ζ. *" The high rate of bankruptcy is a consequence of the high interest rate as much as a cause of it"* (Stigliz and Greenwald, 2003: 145). Thus, the domino effect can arise through the credit channel, that is, an avalanche of bankruptcy may be caused by the diffusion of negative externalities in the network of bank-firm relationships.

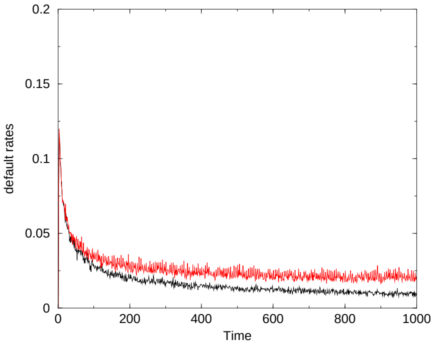

Figure 3: Firms default rates for $\zeta = 1$ (black line) and for $\zeta = 50$ (red line).

This exercise of comparative dynamics leads us to the conclusion that the liquidity is fundamental to avoid shock in the credit market. The increase of interest rates and their consequences have been generated by the rigidity of the supply of liquidity and, particularly, by not to have adapted deposit to the parameter ζ.

Obviously, also the firms rationing rates increases with ζ, as showed by picture (4). Once more it is worth remembering that the deposits are exogenous variables in the model. However what is clear is the importance for the credit market to collect all

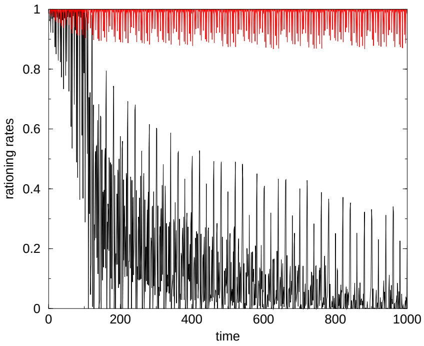

Figure 4: Firms rationing rates for $\zeta = 1$ (black line) and for $\zeta = 30$ (red line).

money in the system: there must not be circulating money in EURACE. The reason is the necessity of the system to be liquid; if the supply of liquidity, of which deposits are part, are not enough to satisfy the demand of credit we will observe the phenomena happening when ζ is "too high".

Now we would like to focus on some characteristics of bank sector such as equity and supply of liquidity.
The next graph (5) shows the average equity and supply of liquidity of banks. The equity rises increasing the demand of credit, however when ζ becomes 'too high', that is, when the demand of credit increases without modification in the supply of liquidity, then the equity does not rise anymore. The reason is that banks do not have enough money for satisfying the demand of firms. When, for instance, $\zeta = 30$ the average supply liquidity of banks oscillates around zero as showed by the red curve of the graph (5), right side.
Once more, so, the role of the deposits and supply of liquidity is clear. It is the crisis of liquidity to generate distortions in the system.

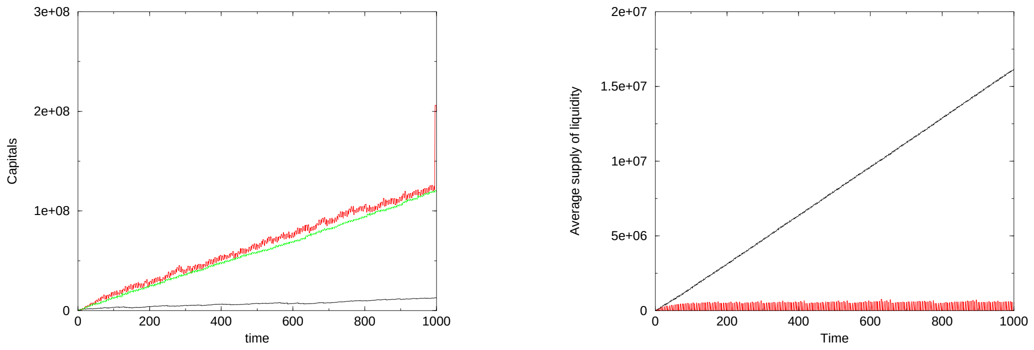

Figure 5: Average equity for $\zeta = 1$ (black line), $\zeta = 30$ (red line) and $\zeta = 50$ (green line)(left side). Average supply liquidity for $\zeta = 1$ (black line) and $\zeta = 30$ (red line), (right side).

4 ***FLAME*** implementation

The differences of this implementation are not in the mathematical structure of the model, but in the organization of the code according to X-Agents theory and in the integration of Credit Market with dummy versions of the other *EURACE* model modules. In particular, we use dummy models of the 'Firm Financial Management Role', the Government and the Central Bank. Moreover, we introduce a Dummy Agent playing the role of those markets not directly interacting with ACM. This implementation is a preliminary step to achieve the complete integration among the *EURACE* models. The goal is to have a stand-alone credit market able to permit us to debug the code and make economic analysis. Since we want to test only the credit market, this isolated ACM allows us to overcome all the problems related to the complexity generated by the integrated model. Consequently we can focus on agents' functions and on the interfaces between ACM and the other strictly related modules.

In X-Agents framework each agent is characterized by a set of states, functions and

messages. The states define the time structure of the model and suggest the function the agent has to activate. The functions represent the decisions of agents and their corresponding actions. Functions may receive messages from agents as an input, and send messages to agents as outputs. The messages bear information between couples of agents in the system. This information is then used as a tool for the action.
The picture (6) displays the FLAME structure of the model, where ovals depict states, squares functions and green lines messages.
In order to understand how the ACM works, in what follows we describe the characteristics of functions and messages.
In the first step, firms activate the function '*firm_ask_loan*' sending the message '*loan_request*' to banks: this message contains the amount of needed loan and the firm financial condition the bank needs of. Using the information included in this message, the bank activates the function '*decide_credit_conditions*' and sends back to the firm the message '*loan_conditions*' with the proposed interest rate and the amount of granted loan. The firm receives messages from different banks and choose the one offering the lowest interest rate: this action is performed by the function '*firm_get_loan*', which sends to the chosen bank the message '*loan_acceptance*'. This message is picked up by the selected banks through the function '*bank_give_loan*', which updates the bank memory variables. These variables are essential to define the relationships with Central Bank and to asses the credit system risk. In particular, when a bank gives loan its credit risk and its total credit increases, while its liquidity decreases. As described in the section **??** banks without sufficient liquidity can ask money to the Central Bank sending the message '*central_bank_account_update*'. Central Bank has to lend if the asking bank has no debts with it. Otherwise Bank does not receive money and it can not, thus, grant all the requested money to the firm. The other condition that constrains bank in its lending activity is the credit risk, given by the sum of its clients' risks.
The borrowing firms have, each month, to pay back interests and installment. Without dealing with the Firm Financial Managnent Role, described by GREQAM, we have three possible cases:

1. the firm has enough resources to pay debt commitments, interests and operating costs;
2. the firm has enough resources to pay the amount of interests and istallments but not operating costs, so it enters in a financial crisis state but not in bankruptcy;
3. the firm has not enough resources to pay debt commitments and interests, and goes into bankruptcy.

In the first two cases the firm is able to pay interests and installments, sending to its creditors the message '*installment*' containing the interests and istallment value. In the latter case the firm sends a '*bankruptcy*' message with the share of refunded loan. These messages are received by banks using the function '*bank_receive_installment*', which updates the bank memory variables such as its liquidity, profits and credit risk.
It is important to stress that all the liquidity is concentrated in the bank system and agents are not allowed to retain money in their pockets. Thus, at the end of each day,

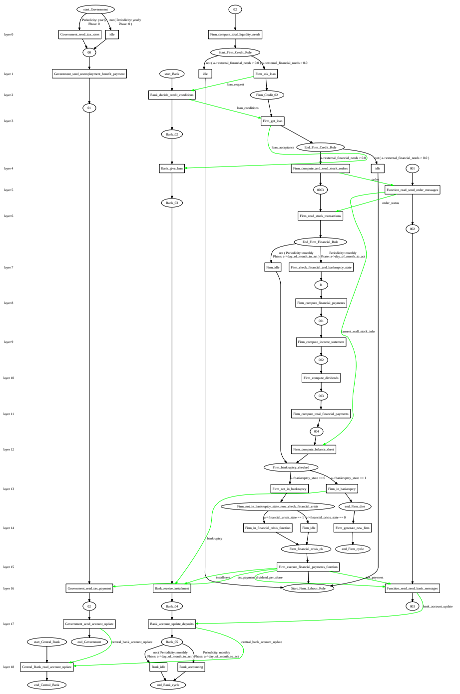

Figure 6: Dependency graph of FLAME stand-alone ACM.

all the transactions made in the system by firms and households are comunicated to the banks by the message '*bank_account_update*' (actually sent by the Dummy Agent) and are accounted for by banks with the function '*bank_account_update_deposits*'. This function updates deposits. The banks in deficit of liquidity always ask money to the Central Bank since an interbank market is not modeled. It is important to remember that the Dummy Agent is substituting for the missing modules. It permits us to collect the lost messages and to send messages coming from the missing models.
All the bank functions described above are activated on daily basis. The only function activated on monthly basis is '*bank_accounting*', which updates the balance sheet and the behavioral parameters such as the γ described in the equation (4).

Agent	Function	Read Msg	Sent Msg
CB	central_bank_read_ account_update	central_bank_ account_update	-
B	Bank_decide_ credit_conditions	loan_request	loan_conditions
B	Bank_give_loan	loan_acceptance	loan_conditions
B	Bank_receive_installment	installment	-
B	Bank_account_ update_deposits	Bank_account_update	central_bank_ account_update
B	Bank_accounting	-	-
F	Firm_ask_loan	-	loan_request
F	Firm_get_loan	loan_conditions	loan_acceptance
F	Firm_compute_balance_sheet	current_mall_stock_info	-
F	Firm_not_in_bankruptcy	-	-
F	Firm_in_bankruptcy	-	bankruptcy
F	Firm_in_financial_crisis_function	-	-
F	Firm_execute_financial_ payment_function	-	installment, tax_pay._divid._per_share, tax_payment
D	function_read_send_ bank_message	tax_pay._divid._per_share, tax_payment	bank_account_update

Table 1: Table of functions per agents. CB: Central Bank, B: Bank, F: Firm, D: Dummy Agent. Not described functions are not correlated with ACM.

4.1 Results

Since the theoretical consistency of our Credit Market model has been checked in the *C* implementation, in this section we perform a simulation with the *FLAME* only to test its correctness under a software point of view. In particular, we want to verify whether our ACM module is well interfaced with the 'Firm Financial Management Role' module and, consequently, ready for the final integration of the *EURACE* model.

The simulation consists of fifty iterations, that is 50 days. The network is composted by 100 firms and 10 banks. Households are replaced by the Dummy Agent.

Every month firms accomplish two main actions: they ask for loans and pay debt installments, inclusive of interests. The demand of credit occurs once per month. All firms, for sake of simplicity, are activated in the first day of each month. Interests are payed back every day after the loan has been taken out. The more relevant initial conditions of firms have been set as follows: *equity* = 100, *total debt=0*, *cash* = 100 and *total assets=0*.

As far as banks are concerned, they have the task to lend money and fix interest rates whenever required by firms. Then, banks collect interests and installments payed back by firms increasing, so, their equity and cash. At the end of the month, banks with positive profits will pay taxes and dividends.

Relevant bank variables have the following initial values: *cash=1000*, *total credit=0*, *equity=1000*, *debts versus BCE=0*, γ_t=0.2, γ_{t-1}=0.4, *BCE's interest 2%*, α=0.8 and $\pi_{t-1} = \pi_{t-2}$=500.

Figure (7) shows the average bank equity along the simulation time. In the first twenty days it increases thanks to interest payments. In the first day of the second month (21^{st} iteration), banks pay taxes and, so, the equity shrinks. Then, since banks grant new loans and, consequently, new interests are payed, the everage equity increases again and so on. It is worthwhile to note that in the second month equity grows less because banks are short of liquidity and lend less money. This is because we have not considered deposits from households (which are created in another *EURACE* module); additional liquidity, disposable for new loans, comes only from firms' payment of installments relative to previously contracted debts.

In figure (8) we can see the average bank cash. After the first period it decreases abruptly because of loan demands occurring in the same time. In the following days, banks receive installments and can build up their liquidity again. The same reasoning for equity applies: we do not have deposits, so cash can only re-establish slowly and is immediately absorbed by new credit demands: that is why the average cash shows along time only small increases.

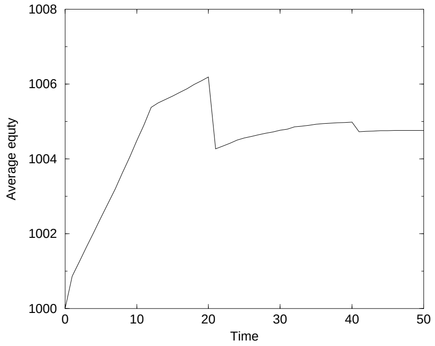

Figure 7: Time series of average bank equity. Jumps occur when taxes are payed.

5 Appendix: On the size Distribution of Banks

In what follows, we sketch a brief guide of stylized facts, concerning the size distribution of banks, usable as benchmark in order to validate and/or calibrate the integrated EURACE model.

The empirical studies on the size distribution of Banks show that data can be fitted with the lognormal and Pareto distributions[9]. These distributions are utilized because they are commonly used to describe skewed distributions and frequently have been used to describe firm size distribution.

The banking literature has long been interested in the size distribution of banks. However, while recent studies, including Berger, Kashyap, and Scalise (1995), Ennis (2001), Jones and Critchfield (2005), have long noted that bank size distribution is skewed, they have not typically tried to fit this using the previously mentioned category of distributions - lognormal and Pareto distributions- . A research that is close to the

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[9] A random variable is lognormally distributed if the logarithm of the random variable is normally distributed.

In economics, the Pareto distribution is a power law distribution often used to describe highly skewed data. It takes the form $f(x) = cx^{-\alpha}$, where $x > 0$ and $c > 0$.

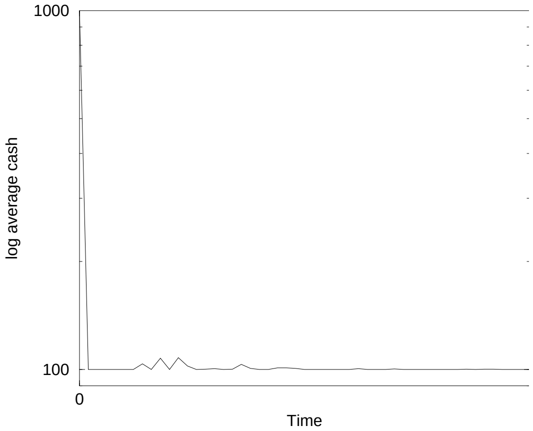

Figure 8: Time series of average bank liquidity supply.

purpose to fit the bank size distribution is that of Janicki et al (2006). As they demonstrate, the lognormal poorly fits the upper right tail of the size distribution. The Pareto fits better with this part of the distribution, but the quality of the fit is poor outside the tail.

An other debated issue is to test if Gibrat's Law holds for Banks. This law states that firm size growth is independent of firm size. This result is important because it ultimately determine the distribution of bank/firm size. According to Gibrat's Law, in fact, to fit the distribution of firm size with a lognormal, two strong assumptions have to be respected: i) the number of firms is stationary and ii) the rate of growth of firms is given by an i.i.d random variable independent of firm size. If one is willing to accept these assumptions as providing a reasonable representation of the evolution of a particular industry, then one can expect that the distribution of firm-bank sizes will converge to a lognormal.

Independently of the econometric debate on the "true nature" of the banks/firms size distribution, one has to take into account such empirical observations, particularly that the bank distribution size is highly skewed to the right, that is, there are many small banks and a few large ones. Figure(9) shows the distribution of assets, which could be considered a measure of bank size, for 1960, 1980, and 2005. This figure demonstrates that there are large number of small banks and a few larg banks.

Once more what is important is, not the bank size distribution, but the skewness of the right tail.

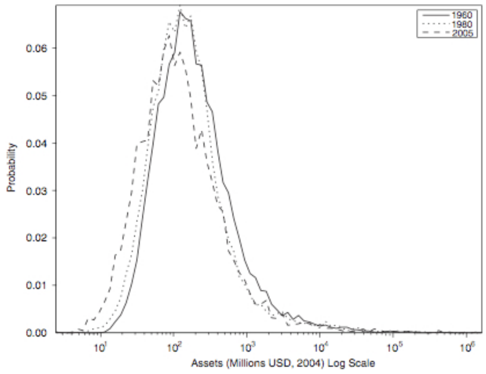

Figure 9: Change in bank size distribution over the time in the USA market. Source: Janicki et al (2006).

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